THE GROWTH OF LOGICAL THINKING IN SCIENCE DURING ADOLESCENCE

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By the same author:

1. Survey of Physics Teaching in Rajasthan (In collaboration), 1968.
2. Planning for Science Teaching (In collaboration), 1968.
3. Problem Solving in Science, 1968.
4. Strategies in Science Education (Co-edited), 1970.
5. Some Aspects of Piaget's work and Science Teaching, 1971.

*6. The Impact Science Teaching, 1971.

7. Recent Trends in Technical Instruction, 1971.

*8. How Children Discover Knowledge, 1974.

9. The Individually Accelerated Science Teacher Education Project (In collaboration), 1975.
10. Project Investigations and Essays in Science (Edited), 1976.

*11. Reshaping our School Science Education (Co-edited), 1976.

12. The Emerging Psychological Frame of Reference for our School Science (Completed), 1978.

*Subsidised by the Government of India through the National Book Trust, India for the benefit of Indian students.

The Growth of Logical Thinking in Science During Adolescence

NARENDERA VAIDYA
Professor of Education
Regional College of Education (N.C.E.R.T.)
Ajmer (Rajasthan)

OXFORD & IBH PUBLISHING CO.
New Delhi Bombay Calcutta

The publication of this Research Monograph has been subsidised by the National Council of Educational Research and Training under the scheme of financial assistance for publication of Ph.D. theses

Published by Mohan Primlani, Oxford & IBH Publishing Co., 66 Janpath, New Delhi 110001 and printed at Bharat Mudranalaya, Delhi 110032

The Mysteries of Modern Science

One of the first men who thought that he had discovered the key to the riddle of the universe was Pythagoras. He found that if a string, when plucked, sounds a C, then a string half its length will also sound a C. So will a string twice as long. If the string is cut into three, then each third will sound a G. If it is cut into five, each fifth will sound an E. On the basis of this evidence, Pythagoras concluded that the harmony of nature was to be found in numerical ratios. He came to believe that if one understood the relationships between numbers one might understand the nature of the world.

It is interesting to look back and consider how Pythagoras used his discovery, and how it served him. He founded a school of philosophy; an elite corps which would protect and care for the discovery, and carry on the work of figuring out the universe. Then he went on to make the discovery which still bears his name, and which—ironically—shattered his dream. He discovered that the square on the hypotenuse of a right-angled triangle was equal to the sum of the squares on the other two sides From this it followed that the diagonal of a square was related to the length of its side by the square root of two: a number which defied expression as a decent fraction. It could not be expressed as the ratio of two whole numbers and was therefore a terrible flaw in the numerical harmony of nature.

The reaction of the Pythagoreans was one of horror. They termed the square root of two an 'unspeakable number' and resolved to treat it as such. They swore never to disclose it, but the secret leaked out, as secrets always do. So much for the first answer to the riddle of the universe.

In a sense, the real achievement of Pythagoras was not the discovery of the magic of simple arithmetic, but the discovery that the universe is a peculiar place and cannot easily be fitted into a pattern of preconceptions, no matter how elegant and aesthetically compelling such a pattern might seem.

Brian M. Stableford

So, when Western education was introduced in India, they also introduced the office of the Director of Public Instruction whose function was to lay down a curriculum centrally, and to see to it that the curriculum was indeed followed, in letter and in spirit, all over the country under the control of the British. The D.P.I. was the master of all education in the province. This was the beginning of standardisation, in the name of standards. The natives had to be better than what they were, this was the altruistic motive. But behind it lay the realisation that education is a very powerful tool in capturing the minds of men and shaping them in the direction that the rulers considered to be desirable and safe. Now, it is interesting to note that over the thirty years since independence, we have not touched this model of a centralised curriculum and a public examination which sets the standard. We had Radhakrishnan Commission, Mudaliar Commission and finally Kothari Commission. These Commissions did very good work and made important contributions to the development of education in India But they did not consider changing the basic arrangements of controlling the mind through education. Books are prescribed, syllabi are prescribed, examinations are prescribed—and all these are done by one authority in a State. We complain that students do not read more than what is prescribed. Students complain that even that which is prescribed is not covered by the instruction in the class. Students go on the rampage on the ground that examinations ask questions from topics which are outside the prescribed syllabi and prescribed books. The tendency all round is to close the minds instead of opening them Instead of education being an experience which frees the mind, we find that education has become a dull routine that stultifies the mind What we have to understand is that we cannot change the situation. unless we realise that a centralised control over curriculum, books and examination goes contrary to the very spirit of creativity, divergent thinking, learning by discovery. learning from working. learning from society, etc. And so, it is not surprising that many thinkers have noted that we have been only tinkering with education when we have been talking of educational reform. The present controversy over the

10+2 structure is a good example. The structure is not so important as the curriculum and even the curriculum is not so important as who controls the curriculum. If education is to make men free, freedom has to be brought into the system of education. I think it is important to realise that it is not just in our country that there is a battle for the mind in the name of education.

Shib K. Mitra

PREFACE

Man is an inseparable part in the evolution of civilisation or humanity. Being human himself, what concerns children, teachers, teacher educators and researchers is of great concern to him for he himself has been a ceaseless inquirer into scientific, technical, social studies and similar multifarious affairs. These reflections over the years have not developed in vacuum for educational ideas like scientific ideas have their remote past in educational principles 'rooted in the wisdom of the ages' traced back to the ancient Greek thinkers like Socrates and Plato. Since then the ideas of many great thinkers representing different disciplines have been woven into the fabric of current educational philosophy and practice. When given this environment, Jean Piaget drinks heavily from this philosophical spring. While doing so, he appears to pay back his ancestral debt abundantly in his own life to the moderns as well as the futurists for it is impossible to imagine today any cognitive study without reference to his work. Whether one agrees or not, a dip in the 'Ganges' of Piaget, does affect one's outlook on problems educational. Why? Because he presents some sort of thought provoking synthesis as obtaining in rationalist tradition of Plato, mental categories of Kant, notion of perceptual change as propounded by Bergson, productive thought of Gestalt psychologists like Max Wertheimer and use of logic for interpretation of thinking (classes, relations, grouping, reversibility and equilibrium) coupled with the cumulative influence of several individual personalities like Calvin, Rousseau, Pestalozzi, Claparede, Binswanger, Bleuler, Rorschach, Meili, Jung and Sechehaye. Philosophical problems continue to recur century after century for philosophers, perhaps in the true tradition of their own beloved subject, pose their problems in unsolvable forms. Piaget, while still remaining a firm speculator, succeeded in stealing part of this territory by tackling psychologically an age old epistemological problem as enunciated by Plato: "How and when can man be sure that his knowledge is true knowledge, when time and again he finds that what he took to be

knowledge has proved to be error?" In this act, he not only kicked aside the fairly well established scientific procedures by establishing his own METHODE CLINIQUE but also constructed his own symbolic logic for interpreting intellectual operations as they develop from the very early childhood to late adolescence. He thus showed how a child, nay a universal child, goes about the business of constructing his own house of knowledge. Continuously bitten by this problem over a period of forty years, he came up with this grand hypothesis:

Development is continuous not only within the individual but throughout all evolutionary levels. From the biological to the social to the intellectual levels, the unity of nature is preserved. The functioning of the loveliest mollusk is based upon the same fundamental processes as that of an Einstein.

Thinking is a 'ghost-like' activity for it has to be inferred rather than observed. In literature, it has been referred to as 'abstracting, analysing, comparing, deducing, defining, discriminating, estimating, generalising, guessing, imagining, judging, knowing, opining, reasoning, recalling, recognising, reflecting, remembering, searching for conclusions and understanding.' While going ahead, one visits the territory of concept formation, it again turns out to be a vast area of investigation in its own right having strong links but, at the same time, not identical with such complex psychological processes as thinking, learning, problem solving, language acquisition and symbolic representation. If this journey is further continued, one enters the complex areas of problem solving, creativity, critical thinking and originality where one is caught into several definitional difficulties. One is very lucky, if not lost, for every milligramme of information obtained, there is a quintal of effort to expend. And here the distinction between original sense and original nonsense disappears. Not long ago, the nuclear scientific Oppenheimer aptly remarked, "It is the business of science to go wrong." Difficulties, therefore, for the cognitive psychologist multiply several folds and for achieving success, problems have to be posed more and more productively in the phraseology of Gestalt psychology with a view to investigate the highly varied complex processes of thinking which may ultimately lead to the development of understanding, generalisation, discrimination, concept development and attainment. The problems do not end here because the researchers and the practitioners do not invariably ask the same question. These split questions and

possibly the consequent split answers are bound to appear in the business of any science, educational psychology, being no exception. The workable solution under these circumstances lies in considering the two in isolation as is frequently done while managing anamolies in science. The basic problem here is to relate the most powerful concepts of science to the mental development of children whose answer goes a long way in providing the psychological structure to any one of the school subjects. Thanks to the recent efforts of Ausubel, Bartlett, Beard, Bruner, Flavell, Gagne, Guilford, Hans Furth, Hearnshaw, Humphrey, Inhelder, Lovell, Lunzer, Peel, Kraplus, Piaget, Schwab, Skinner, Suchman, Vernon, and Wallace, the problem-territory in this decade has been exploded and consequently, the problems in pieces, like the discovery of fundamental particles in atomic physics, lie more in the zoo rather than in the jungle. This was hardly the case over twenty-five years ago. The emerging literature on thinking clearly tells that children learn spontaneously, they acquire schemes of thought regardless of school influences. They learn from each other, and learn equally well in formal situations. At the same time, they also possess first hand knowledge about men; and their affairs in their immediate environment. This does not amount to saying the sun rises in the east or sets in the west. It, on the other hand, means that the child's thinking is not at all a chance or random behaviour. It is sensible, intelligible and predictable against the available theoretical psychological constructs. In this perspective, the field as a whole poses the following problems which overlap each other to a varying extent:

(i) What are, specifically speaking, those conditions under which learning takes place maximally? What exactly is the role of hints and cues either in supporting thinking or in the formation of concepts? To what extent can concepts be downgraded?

(ii) At what age is formal reasoning manifested among different categories of pupils? How is formal reasoning different from concrete reasoning? What conditions determine transition between the two stages.

(iii) How is thinking generalised in each of the two grades? Does it develop in stages? Is it possible to accelerate stages? To what extent is pupil behaviour modifiable? What is the role of planned experiences in concept development and problem solving? Is creative thinking influenced as well?

(iv) What outside variables determine individual differences in

thinking among children? Does sex play an important part here? Do regional differences exist in thinking because of the differing expectations elsewhere?

(v) What really governs or determines going forward in thinking on the basis of incompletely supplied information? Or what exactly is the relationship between concept formation and its onward mobility, for example, application?

(vi) Does 'instruction precede development' (L.S. Vygotsky)? Is learning to take place mechanically or from page to page within the textbook? Or is it possible to save children from subsequent teaching once the initial learning has been fully acquired? Quoting J.S. Bruner, "Is it possible to get a 'maximum of travel' of what we have learnt?" If so, what kinds of thinking processes does training generate under different conditions? Are they chips of the same block or are they different? Are they of temporary or of permanent character?

(vii) What errors children exhibit at various age levels? Are they, psychologically speaking, interpretable? Is it possible to develop a museum of these errors? Is it possible to demolish this museum through intervention? What about the role of maturity here?

(viii) Is learning possible in the absence of schemes of thought in relation to academic subjects taught at school? Do these schemes of thought have any psychological existence or relevance?

(ix) Why children fail to verbalise their concepts or methods of procedures especially when they have, in fact, acquired them?

(x) Lastly and basically, how children coordinate information? What are the underlying mechanisms? Have they strong links with symbolic logic? Are they of any psychological significance as well?

The present work attempts to answer part of the main problem partly by examining empirically the following hypothesis: Thinking processes develop unidimensionally or, in the act of growth, they are linear in character, i.e., they do not follow zigzag paths. Secondly, when individual thought processes are aggregated on the basis of similarity and identity, they constitute schemes of thought which, again, in turn, lie on a continuum. Given the data elsewhere, is the simultaneity of the sequence of development maintained in all children of all people all over the world? In short, using case study approach, the present study attempts to delineate the structure of these schemes of thought empirically, examine their growth during adolescence and fix them mathematically through factor analysis, using Hotelling method. In this

field experiment, 200 pupils (100 boys and an equal number of girls) studying in grades VI to X matched on intelligence and socio-economic status were observed, solving a series of seventeen different problems, which were further analysed into 87 thinking processes and a few open processes which, in turn, were further regrouped into seventeen schemes of thought. The main findings of this study indicated:

(i) Except for occasional fluctuations, average performance on each problem increases with grade. Mean performance in most of the cases favours boys rather than girls. But both boys and girls try hard to equalise their performance as they move into higher grades.

(ii) A given problem, part of the problem or a process in that problem is solved successfully (or failed) over a wide I.Q. range both within and across the various grades.

(iii) A given problem is solved in stages. It is possible to identify stages in the solution of any problem.

(iv) Unexpectedly, pupils commit a large number of errors while engaged in problem-solving. These errors further increase in the higher age groups when they fail to grasp the main requirement of the problem. Among these errors, the dominant ones, shared by more than 15 per cent of the pupils, largely speaking, suffer a hump before they finally decline with increasing grade.

(v) Contrary to Piaget, pupils are not in a position to exhaust or suggest all possibilities, combinations and tests. Over three-fifths of the pupils from grades VI to VIII are badly affected when a problem inheres reversibility in thinking.

(vi) The complex problem solving processes arise from simple thinking processes.

(vii) The role of the nature of the problem being critical in investigating thinking; adolescent pupils, contrary to Piaget, are affected to a great extent by the content of the problem.

(viii) Whereas adolescent pupils are in a position to set up hypotheses, they are not in a position, contrary to Piaget, to test them which shows that their minds have not yet become experimental. Most of the adolescent pupils do not, contrary to Piaget, possess schemes of proportion as well as generalisation to algebraic symbols.

(ix) When a problem is solvable through two schemes of thought, one inferior and the other superior, and if the latter is not well developed, the resort to the former may favour quite a few in solving that problem. In case both are conspicuous by their absence,

there is little chance for the supplied hint or illustration or even analogy to be utilised in solving the problem successfully.

(x) Except for occasional fluctuations, the mean performance on the various schemes of thought shows an increasing trend with grade. When mean and standard deviation are kept at 20 and 3 respectively, the gains in terms of T scores over the five year period are dismal implying thereby that they are characterised by gradualness, slowness and laboriousness as they evolve across the grades.

(xi) Using Principal Component Analysis on the combined matrix, containing forty-five variables and the varimax rotated factor structures with a view to obtain the hypothesised factors and interpret them psychologically, the following multiple factors appeared:

1. Schematic Learning General
2. Adjustment
3. Problem Orientation
4. Sensing Problems
5. Symbolisation
6. Testing Hypotheses
7. Using Constant Difference
8. Aspect Character

(xii) The top group differed from the bottom group on all the five measures of adjustment, understanding of the problem and all the seventeen schemes of thought. They, on the other hand, did not differ significantly from each other in respect of the following variables: felt difficulty of the problem, confidence in the problem and interest in the problem.

At this stage, it is necessary to highlight an interesting finding, hardly looked for, in this investigation. It has appeared in several contexts: supplying an answer already available within the body of the problem, last item needing an arithmetical or algebraic answer; counting the total number of trails or efforts made; returning to the same step after suggesting other steps; and the number of arbitrary and extraneous considerations brought into the problematic situation during problem solving.

Why should it happen? Is it the case of an adolescent playing with figures thoughtlessly or arbitrarily in the hope of being favoured with good luck? Is it his case to respond to the varied test items, regardless of consequences and meanings, in any manner he likes? Is it really the case of lack of seriousness on his part? Is it his case of being caught between the horns of a dilemma and getting murk? Is it the case of hot chase trying hard to choose in haphazard

directions as if in the manner of closing in on the problem? Does it illustrate that mastery of a thought process is through a path: Uphill, thorny and often erratic? Or does the adolescent regress as if on an adventurous Piagetian journey during which he is trying hard to educate himself, thinking that the right path to concept development lies in flourishing on experimental failures? Or is it the characteristic of problem solving situation in which either understanding suffers a dip or errors a hump? Or alternatively, is it an act of rubbing his schemes of thought by the adolescent wrongly, especially, when he has personal reservations about his self acquired knowledge in contrast to school learning which does not set right his firmly held self centred thoughts? It is anybody's guess? In a personal communication, J.S. Bruner, who had encountered this phenomenon earlier and had not followed it, had this much to say:

The type of error that you refer to, which we speak of growth error, is one in which the growing child tries out a new strategy although it is not well developed and uses it in place of an older one which has been working well. It is errors of this sort which suggest to me, the venturesomeness of learning during this early period, that human beings are willing to shift to a less certain more powerful strategy before they have it under control in preference to one which is safe, sound and dull.

Even on philosophical considerations, the essence of human mind consists, not only, in trying but also in challenging the unknown even by digressing in an endeavour where efforts are many but consequent rewards few and far between. If these efforts are regarded as errors: growth or otherwise, as is usually the case, a natural corollary contrary to positive reinforcement in methodology of instruction arises. The teacher in his day-to-day teaching should help his pupils to see the same problem or concept from as many varied angles as possible with a view to form firm concepts having a "broad measure" of generality in a medium where learning from errors on part of his pupils becomes an integral part of his teaching meaning thereby that learning from errors is a respectable educational activity. On the very face of it, it appears dilatory but a stitch in time, as the adage goes, saves nine. But only a longitudinal study is going to establish this Brunerian hypothesis. Returning to the Hump Effect, in Serial rote learning, Hump effect in the form of 'bow' has appeared when frequencies of errors were plotted for a group of subjects engaged in learning each item in the Serial Position Effect. In the context of this study,

the problem arises: Does Hump effect exist at the lower stages of mental development as well? If so (and confirmed), a short tunnel to learning may be obtained which may be theorised upon like the theory of fermentation. If effort in this direction is successful, the needed fill up may usher in the second psychological revolution in learning consistent with the spirit of educational technology especially when the second scientific and industrial revolution is picking up very fast in our country.

The organisation of this book is very simple which contains six chapters. The first chapter introduces readers, in brief, to the impact of science and technology, the nature of science and the assumptions underlying science education programmes.

The second chapter, in three sections, orients the readers to the basic rationale, the research study and the development of the test instrument. The third chapter, again three sections, orients the readers to the basic rationale, the research study and the development of the test instrument. The fourth chapter describes qualitatively the growth of problem solving processes on some selected, problems due to scarcity of space. The fifth chapter, in turn, provides quantitative information on the analysis of thinking processes, schemes of thought and sex differences in problem solving. As usually happens, the work concludes with problems for further research arising, particularly speaking, out of this study.

Acknowledgements are numerous. I had the opportunity to consult the following libraries: Institute of Education, London; Centre for Science Education, Chelsea, London; School of Education, Reading University, Reading; Department of Education, Birmingham University, Birmingham; Institute of Education, University of Leeds, Leeds; Central Institute of Education, Delhi; N.I.E., N.C.E.R.T., New Delhi; Centre for Advanced Study, M.S. University, Baroda; and Regional Colleges of Education, N.C.E.R.T., Bhopal and Ajmer. Whereas N.C.E.R.T. allowed me to utilise liberally a short term Commonwealth Education Fellowship award, the four Principals of the Regional Colleges of Education, Prof. J.K. Shukla, Prof. R.C. Das, Prof. G.B. Kanoongo and Prof. J.S. Rajput provided me every facility in the furtherance of this study. As regards other individuals, I am heavily indebted to Prof. W.J. Jacobson, Teachers College, Columbia University, New York; Prof. J.J. Koran Junior, College of Education, Florida University, Florida; and Dr. Amarjit Singh, Research Officer, School of Education, Reading University, Reading, England who

provided me intellectual and emotional support abundantly whenever needed. I am equally thankful to my guide Prof. L.K. Oad, Principal, College of Education, Banasthali for not only suggesting this problem but also for his very kind and incisive remarks. Also, a word of praise to Mr. B.K. Sharma, Lecturer in Education, College of Education, Kalakankar, who did the additional computations not done by the Computer Unit, University of Reading, Reading, England. I am much obliged to Mr. the heads of government institions at Nurpur, Nagrota Bagwan and Dharamsala in Kangra District of Himachal Pradesh who provided me pupils for intensive testing during the entire summer in 1973. Last but not the least, I am obliged to Dr. G.N. Bhardwaj and Mr. T.S. Sandhu for having read parts of the manuscript.

What is the basic significance of this exercise? It is this: Get aside, for you are obstructing pupil vision by over-teaching and testing statutorily. Precisely for this reason, the student fails to experience fundamental scientific phenomena as well as honest scientific ecstacy or frustration in the face of success or failure. Chances are that he may miss the growth of something edgy or hardiness in his thinking. Consequence: resulting inability even on the part of teachers to ask even the elementary question, i.e., why children come to school? Or are they being really educated at all? Striking hammer on the cold iron will no longer do! Pupils with vibrating minds who master the stimuli rather than the restraints will not drop from heaven. Where to strike, therefore, implies a good knowledge about the nature of mental development in all its highly varied aspects. In Methods Courses, we as a community real or meta need not for long exchange banalities amongst ourselves. In this context, I shall feel my efforts amply rewarded if this work succeeds in stimulating others in relating teaching of school science to the intellectual growth of children, a barren territory not only in the developing countries but also the highly developed countries as well.

NARENDERA VAIDYA

CONTENTS

CHAPTER I

SOME CONCEPTS RELATED TO PROBLEM SOLVING IN SCIENCE

Impact of Science and Technology

We live in an exciting century, which is complex, in its entirety: physically, philosophically, psychologically, scientifically and technologically. The global outlook is changing fast when one sees the present in the light of the past cumulative achievements of human race: the traditional epoch comprising several generations is breaking up fast. Metaphorically speaking, within our own generation, it has experienced, in a traumatic manner, three explosions popularly called, explosions of knowledge, population and aspirations. Imagine the consequences! Distance and time stand annihilated. Atom, electron, nuclear fission, radio physics, biochemistry, radio chemistry, synthetic fibres, newer drugs, discovery of revolutionary building materials and techniques, atom smashers, computers, communication satellites, electron microscope and lastly, the space travel (the highly imaginative physical adventure of man in space) have become household words. They have stimulated our thinking and action profoundly but with little accompanying change in human outlook on a grand scale [1]. Theoretically speaking, man has fully utilised the benefits of the First Industrial Revolution. With this has disappeared the era of making positive statements, a chief distinguishing characteristic of the nineteenth century. The concept of chance, probability and statistical reasoning have entered current thinking and doing [1]. Four multiplied by three equals three multiplied by four, one need not, say twelve, one of the clear lessons of present day modern mathematics [2]. The Second Industrial Revolution, like the proverbial Jin of the Fisherman, is knocking at our

doorstep. U Thant, the former Secretary General of the U.N.O., stated the global implications of new Science and Technology for us to ponder:

> The truth, the central stupendous truth, about developed countries today is that they can have...in anything but the shortest run...the kind and scale of resources they decide to have....It is no longer resources that limit decisions. It is the decision that makes the resources. This is the fundamental revolutionary change...perhaps the most revolutionary mankind has known [3].

But at the same time, there is the worst fear and pessimism overtaking us all that we may not live to see and invent personal, collective and global bright futures. This is despite the inherent eighteenth century 'plus' point where in the philosophies comprising philosophers, scientists, statesmen and other men of letters and critics knew firmly that 'the commonwealth of learning was older than the commonwealth of political nations' [4]. The same sentiment was well epitomised in the Father of Our Nation, M.K. Gandhi: 'I want the cultures of all lands to be blown about my house as freely as possible' [4, 5]. This is, simply, a highly disorganised situation, partly transparent, in which we find ourselves. It tends to threaten increasingly both man and his abode. In fact, it is a much more intense nervous situation both qualitatively and quantitatively than the one encountered a few generations ago when every scientific discovery evoked an emotional reaction of a varying degree. Society, however, accommodated to each for each discovery or invention was followed by a comparatively longer period of consolidation until another one appeared on the scene. This situation is now no longer true for the consequent gap between a given scientific discovery and its industrial application or commercial exploitation is being drastically cut down by the highly industrialised nations [6]. The very texture of questions in our times, according to Kingsley Dunham has changed: 'Can our species survive the next few hundred years' [6]? why? Because a disturbed and destructive atom along with an erratic computer under political misadventure may explode the whole world to its absolute wretchedness within a flash of split second, hitherto unthought and undreamt of by humanity during its entire march to present day progress since recorded history. No state, rich or poor, developed or developing, can afford to remain in a permanent state of disequilibrium. For example, even in England, a feeling is growing that the 'plus' factor of their educational system is fast

becoming an outmoded symbol when the words of the *Observer* ring:

> The main reason that we are having such trouble coming to terms with the 1960's, is that the education in this country is insufficient and out of date....What as a nation we cannot afford is to go on being undereducated. In the economic sense, the expenditure is an absolutely indispensable investment in national efficiency [7].

The problem is, in fact, more serious as is borne out by documents like Policy Statement in Science Education; Education and the Spirit of Science; and the Report of the Kothari Commission, which surveyed nearly the entire educational system of our country, not long ago [8]. In the context of new 10+2 curriculum, the recent publications of the National Council of Educational Research and Training, New Delhi tell more or less the same story [9]. The lesson to be learnt from the above mentioned few documents is clear. The present day education which prepares the embryo citizens must not remain pre-scientific and pretechnological in its dispensation [6]. At the same time, it must also exert a liberating influence on educators as well. Paulo Freire in "Unusualld eas About Education" goes a step still further when he says that the "educator must die as exclusive educator of the educatee in order to be born again as educatee of the educator" [10]. When seen in isolation, it appears to be an 'unfinished business' in the ultra modern scheme of education suiting the latter half of the twentieth century. It is only then that it becomes the 'Age of Total Man, Man Entire and All of Man' in the words of Faure Edgar [6].

In summary, it is becoming increasingly important to formulate and reformulate problems arising out of rapid strides made in the field of science and technology with a view to solve our age-old problems of food, shelter and clothing. But this is not enough. Scientific values along with their associated modes of thought need to be very widely fostered among the masses through the medium of education, of course, of right content and form. One then need not worry about the existing gap between social and natural sciences. Under the right set of open circumstances, Marx, not long ago, foresaw this eventuality: "The natural sciences will one day incorporate the science of man, just as the science of man will incorporate the natural sciences;" there will, be then a single science [11]. About eight years ago, a tiny document on Education and the Spirit of Science, published by the National Education Association of America, stressed the same point, that is a 'general worldwide

fostering of the spirit of science' [8]. It is then not difficult to see the relevance of age-old Hindu, Muslim and Christian scriptures: 'simplicity, austerity and charity' as one of the best ways to save mankind [6].

Assumptions underlying Science Education Programmes

Within the last 16 years or so, revolution in science, both in content and form, has occurred in different countries of the world for different reasons. PSSC; Chem Study; CBA; BSCS; SMSG; Science A Process Approach and Nuffield Science Teaching Projects have become world known curricular programmes. Both the developed and the developing countries are in the nervous grip of their own internal and external problems as a result of this revolution. Both are trying hard to reconstruct their educational systems afresh [12]. It is, therefore, difficult to mention in its entirety, all the basic assumptions underlying effective science education programmes suiting varied ends. But the following, however, appear to be quite relevant for us in this country.

1. Every normal child encounters several diverse problems. A few of them, he regards very fundamental, significant, worthwhile, relevant and meaningful in the light of his personal needs, capacities, capabilities, interests and past experiences.

2. Most of these problems when collected and classified for a given group of children appear to stem from significant areas of human living.

3. Children remain fundamentally alive to their proximate environment through self-explorations, rich experiences, directed observations and even the practice of control experiments. On further analysis, common reaction patterns for a given group of children appear to emerge despite characteristic individual reaction patterns.

4. Given the free climate where children commit intelligent as well as useful mistakes, they tend to make effective and intelligent use of their evolving concepts in discovering new knowledge through the medium of the various processes of science. They can, thus, be easily stimulated to:

(i) Phrase and rephrase pinpointed problems for investigation at depth;

(ii) Suggest sharp hypotheses, test some of them against the given data; and also offer a few likely explanations;

(iii) Set up control experiments with a view to distinguishing clearly between relevant and irrelevant variables;

(iv) Draw reasonable conclusions from control experiments;

(v) Think out and suggest likely reasons for any conflicting evidence, if came on the way; and

(vi) Feel the sense of achievement and purpose.

5. Unlike adults, psychologically speaking, children do not appear to suffer from any 'fixedness' in idea or approach. They can see the same problem afresh without suffering any discomfort or loss of face. They continue to improve and refine their semi-scientific concepts and skills through personal experiences, others' experiences, thinking, doing, reading, writing and improvising during project work.

6. Children tend to follow their teacher. It is this role of the teacher which has become suspect in the wake of Revolution in Science Teaching. He should, therefore, demonstrate to them the various elements of scientific methods by self example through highly diverse problem solving activities. In short, a key framework becomes available in which the teacher introduces youngsters to the gradual mastery of research operations [13].

Any effective programme of science education should rest on most of the above mentioned assumptions. When observed in action, even partly, these assumptions also become the characteristics of effective science education programmes. It is possible to clarify them still further through additional research in different types of schools, rural and urban. According to Mitchell, discovery comes naturally to the vast majority of children at all ages. 'Essay in discovery' is not at all foreign to their nature. In his opinion, it is entirely the fault of the adults, if the children do not show this inquisitive behaviour for 'curiosity about how things work is one of the strongest drives of young children' [14].

Why This Study?

It is necessary to carry out such studies because they generate firm knowledge in the area of methodology of instruction. Secondly, there is a widespread impression accepted unreflectively that there is something basically wrong with the present day learning of teaching of science. It is alleged that it suffers from several defects, chief among them being:

(i) It is bookish and consequently examination bound in character. Method of approach is, gererally speaking, oral.

(ii) In its day-to-day business, it ignores the real nature and spirit of science.

(iii) It is approached mechanically like filling in empty pots because the teacher is always in a hurry to cover rather than to uncover the syllabus.

(iv) It ignores the development of functional knowledge, acquisition of skills, attitudes and appreciations.

(v) In short, science students hardly make use of their 'talents and tools' while learning and acquiring scientific knowledge and skills meaning thereby that they fail to apply their knowledge to new, novel and unknown situations [15].

Firstly, this criticism is based upon untested evidence. Secondly, according to Gestalt psychology and Geneva school, adolescent pupils show a wide variety of intellectual behaviours, while confronted with those problematic situations, which do not require any specialised knowledge for their solutions. The basic notions underlying these two systems, apart from others available in stimulus theories, have varying degrees of relevance for education of adolescent pupils. The reason is simple: education for understanding and problem solving is gradually becoming the chief goal of instruction, in our times. Thirdly, several problems in the field are crying for solution. Examples are: the workable definition of thinking and problem solving; role of past experience in problem solving; and the determination of functional relationships between problem solving and personality variables. Fourthly, if one analyses the basic problem at depth, several sub-problems and difficulties appear which need to be surmounted before one can understand clearly the nature of problem solving, learning, thinking, and task environments.

The difficulty mainly lies in failure to understand the sequence of reasoning from early childhood to late adolescence, the functional relationships between the various characteristics of problem solving and varied problem solving processes under different conditions of administration. Currently, this difficulty is further aggravated because there is no commonly agreed definition of the very word 'problem' itself. Fifthly, new tactics and strategies, it is suspected, not only enhance learning but also develop independent learning styles. Sixthly, it is further hinted that the 'efficacious study does not depend upon the scientific rigour of the teaching, the value of which is in any case partly illusory' [6]. If true, this necessitates a fundamental shift in the role of the conventional teacher in his day-to-day classroom teaching. Hardly any reference so far has been made to educational technology to solve any instructional problem. If the goal of life-long learning is accepted, then even the

conventional school stands threatened, for it will 'gradually become a club, a workshop, a documentation centre, a laboratory and a place of assembly' [6]. Then why not accord to the city the role of the Best Educator? Seventhly, it is possible to investigate thought process in terms of S-R theories, Gestalt school, Geneva school, Computer science and Accelerated learning. Other outside variables can also be added for further clarification.

So here fundamental research is necessitated. The investigation of thought processes is, therefore, a three-tier problem, involving research on thought processes, methods of teaching or influence of varied training procedures and selection of basic concepts for teaching-learning process. Naturally, a very broad frame of reference is needed to answer these questions, which can only be built if there were a good number of specific studies, at least, in each of the above mentioned areas. The present investigation, as the title of the study shows, centres on the investigation of thought processes of adolescent pupils. It attempts to advance understanding of certain aspects of thinking with the help of problems each inhering a continuous chain of reasoning, especially designed for this study. It also attempts to determine relationships, if any between certain aspects of thinking, included in this study and some outside variables like intelligence, various immediate reactions to the problems on presentation and adjustment.

Concluding Statement

Science and technology have become the vast 'growing edges' of our society in which several dimensions intervene—philosophical, psychological, sociological, political, scientific, technical and local. When one picks up any one of the obvious trivial problems for investigation at depth, the above mentioned dimensions appear knowingly or unknowingly on the scene. These impinging variables make the general solution of problems a bit distorted. It, therefore, becomes imperative for any nation, developed or developing, to reflect systematically on all the interconnected problems in an open (or productive) frame of reference. There is another reason for taking this step as well. Men of vision of the present generation are saying firmly that the present status of knowledge has ceased to be a reliable and valid guide to future actions [6]. Physics prospered, unexpectedly in the twentieth century when a verdict of death was announced on it in the closing decade of the nineteenth century by the then great philosophers and scientists [16].

So reflection can pay off either way in the long run. "Not to Do This is to Court Mediocrity," according to Professsor W.J. Jacobson [17].

Speaking restrictedly, the basic ideas underlying S-R theories, Gestalt psychology, Geneva school and Accelerated learning and teaching have varying relevance for us in this country as well. Further, if effective teaching is defined in terms of learner learning or teaching means forming learning situations (one of the vertices of the evaluation triangle) in which pupils explore the environment, invent concepts and apply them in several diverse problematic situations (the philosophy underlying the Science Curriculum Improvement Study), then his role is to undergo a fundamental change in the conduct of our present classroom teaching [18]. The depth of focus, indeterminate by itself, with the passage of time, will be placed within the brackets of concept-formation, problem-solving (assembling included), self-learning and maintenance of life-long education in an increasingly getting loaded scientific and technological society. What is being hinted at? It is this: no one's education will be complete in his life-time like the meeting of parallel lines. Fortunately or unfortunately, we will be coming closer to another proverbial belief in another context: The King is Dead. Long Live The King ! Margaret Mead goes a step ahead when she says:

> We must place the future, like the unborn child in the womb of a woman, within a community of men, women, and children, among us, already here, already to be nourished and succoured and protected, already in need of things for which, if they are not prepared before it is born, it will be too late. So as the young say, 'The future is now' [19].

A grading educational system like the one in our country degrades itself. In this restricted sense, this means that we have to be increasingly aware over the years of what it means to teach and learn science scientifically, a much oft quoted slogan in the newly emerging literature on science education—both at home and abroad [20].

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CHAPTER II

THE STUDY

SECTION A: THE BASIC RATIONALE

General Orientation

It is deemed necessary to make a few remarks in regard to the general orientation of this study because this not only commits the investigation to a particular viewpoint but also orders and disciplines the data accordingly in the phraseology of Fred N. Kerlinger [1]. This is despite the fact that every research worker has at his disposal several research designs, methods and procedures for collecting, quantifying and analysing data out of which he can pick and choose any one, on the sole principle of congruence, that is, appropriateness to the research problem at hand [2]. The very decision: "how to operate on the problematic situation", is a function not of the research design but that of the researcher alone, for the main objective is to meet the proposed aims and objectives of the study; and hence, the need for clarification. Mathematising to a certain extent, later on, then becomes the essence of any experimental science. Let us illustrate psychometric versus developmental aspects of intelligence.

Firstly, it is widely known that factor analysis, a highly mathematical technique, is used to determine the minimum number of factors underlying several variables which influence any given phenomenon: psychological or educational. Its second purpose to test advance hypotheses about the relations existing among variables is little known and little accepted [1, 2]. It is in the latter context that factor analysis has been used in this study. Secondly, it is important to refer to the two aspects of intelligence which started with a

common ancestry. Over the years, intelligence has been investigated by two complementary approaches, each undertaken independently of the other, largely speaking, in the "Holy Empire of Correlational Psychology" whose two princes being "psychometry and development" [3]. To put in other words, it means that the central problem of intelligence has been tackled from these two stand points, namely, 'measurement' and 'finding sense before measurement'. Consequences: differing orientations to studies, methods and procedures, modes of sampling, collection of data, varied analyses and interpretations of data, use of parametric and non-parametric statistics and the application of logic in the analysis of data came handy to the respective workers engaged in this field. It is not at all the intention of this chapter to make judgements on the superiority of one approach over another. After all, what question to put is a matter of individual choice [2, 4]. It is the quality of the obtained answer in relation to the proposed aims, finally speaking, on which the technique turns rather than the very sophisticated nature of the very design itself. It hardly matters if the answer turns out to be wrong for it can be, later on, contradicted and improved upon [4]. For example, the once infinite considered velocity of light by Aristotle was measured after hundreds of years [4]. There is always then the resulting progress on account of the advancing scientific wave front [4].

Both approaches—developmental as well as psychometric—recognise cognition as the fundamental variables which requires to be investigated first at depth. The other two variables, namely, 'hot cognition' (personality traits) and 'will' can await. Secondly, both value highly experimental methodology while investigating their respective problems. Thirdly, both acknowledge the existence of maturation as an insignificant construct in their explanations. Fourthly, both agree on the essence of intelligence: rationality; and individual reasoning and the formation of mathematical structures respectively. Fifthly, when the respective findings are consolidated and reflected upon, the resulting concept of intelligence is enhanced in respect of the individual description as well as clear separation of individual abilities [5]. At the same time, it is paradoxical that the same behaviour is described as well as assessed in different terms. According to David Elkind, these differences arise due to the 'type of genetic causallty they pre-suppose, the descrip tion of mental growth they provide, the contribution of nature and nurture they possess and finding interindividual and intraindividual differences' [5].

Let us first take the psychometric or the statistical concept of intelligence, I.Q. turned out to be the most powerful concept which worked excellently well for it achieved that it was supposed to achieve [6]. However, as usually happens, this new term came to be criticised over the years despite the fact that, in the beginning, its criticism was restricted to talents such as 'art, music and dramatic ability'. Following the footsteps of Thurstone and using a battery of highly imaginative tasks coupled with advanced statistical analysis, Guilford and his associates suggested three basic dimensions to the structure of intelligence, namely, contents (N=4); operations (N=5); and products (N=6). Like Mendeleev, they proposed theoretically a sort of periodic table for the structure of intelligence containing one hundred and twenty (4×5×6) factors or human abilities [7].

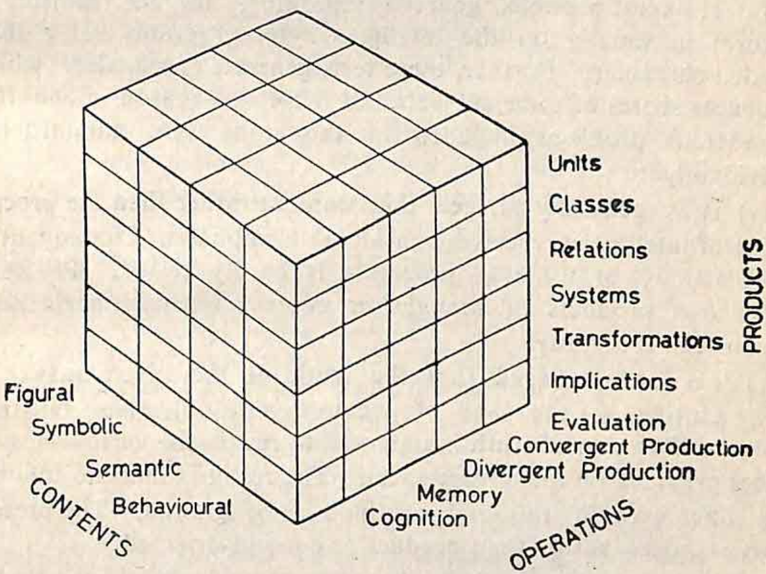

Fig. 1. The structure of intelligence as propounded by Guilford and his associates.

The criticism on I.Q., however, still grew louder and louder in the wake of new knowledge. New experimental concept appeared. It was 'creativity' won from mute controversies arising out of conflicting evidence resisted by the Age of Terman. Intelligence, when seen psychometrically, is said to suffer from the following defects.

(i) It stresses, strictly speaking, conventional values of academic importance, memory also included. Emphasis, largely speaking, is on measuring the various aspects of convergent thinking. About seven years ago, Raymond Cattell challenged this assumption and propounded the mathematical existence of two forms of 'g', namely, crystallised intelligence (conventional) and fluid intelligence, a sort of creative behaviour characterised by 'adaptation to new situations' [8]. It is for this reason alone that a few open problems have been included in this study.

(ii) It is not known how the various test items for any intelligence test come out to be finally selected. The only basis for their inclusion appears to be empirical, that is, they have worked successfully in the past. In place of conventional test items, problems inhering a continuous chain of reasoning have been included in this study.

(iii) It is not possible, generally speaking, for the subjects to perform maximally on the intelligence tests, reason being their speeded character. Further, these tests generate test anxiety which influences scores of some subjects. It is for this reason alone that the various problems included in this study were administered individually.

(iv) It is generally alleged that content rather than the process aspect of intelligence receives considerable emphasis. Consequently, the plasticity of thinking processes is hardly tested. Processes rather than products of thought or content receive considerable emphasis in this study.

(v) It is further alleged that the truth of the given answer is over-quantified for the sake of accurate and objective statistical analysis. This sort of truth is supposed to rest in the various answer spaces provided on the answer sheet. The result is that the qualitative differences in thinking continue to be ignored. The present study is process rather than product or content-oriented.

(vi) It informs fairly well about the rate of mental growth but tells very little of what is actually developing or growing. The present study, largely speaking, attempts to trace the various thinking processes right from the moment they appear to emerge to the moment they appear to be well established.

(vii) It is often said that the very test format used in intelligence testing is quite inadequate. It is true of the multiple choice variety test items, considered to be extremely effective in achievement—testing (objective based testing). Why? When intelligence

is assessed metrically using this form of testing, it 'penalises those who perceive subtle points unnoticed by less able people, including the test makers'. Further, disciplined expression and other aspects of creative thought hardly attract any attention [6]. The present study attempts to rectify the above mentioned deficiencies of intelligence tests by selecting highly diverse problems, each involving a continuous chain of reasoning, which, on further analysis, can be classified into schemes of thought, on the principle, that all those processes which appeared to belong together were put together regardless of the typology of problems.

I. A. Taylor, severely, criticised the so called advantages of intelligence tests by saying that it was not only a 'product of the western mind' but also it essentially engaged itself too much with solving out of routine insignificant or trivial problems successfully at top speed. Other cultures, on the other hand, may encourage something different: solving of highly stimulating problems valued highly by 'making all the errors necessary and without regard for time' [9]. It is a mistaken calculation, quite avoidable, when one lumps together talent, creativity, and conformity; and raises it to the status of either a numerical figure or an index representing the 'sum total of man's mental functioning'. Moreover, it is highly dissatisfying even from the philosophical angle for it constitutes, according to Calvin W. Taylor, an 'insult to the brain, to the human mind, and to the human being' [10]. Peter M. Bentler appears to have closed this issue when he says:

> As is well known among psychometricians, but little popularised among test constructors and users, the multicategory and binary scoring schemes used in most psychological tests hinder the effective use of factor analysis in discovering dimensions underlying the interrelations of variables. Indeed, when the measurement scale is ordinal, and one has confidence only in the rank order of values of a variable, the theory of reliability (E.G. Gulliksen, 1950; Lord and Novick, 1968) and factor analysis (e.g., Harman, 1967; Horst, 1965), as well as other methodologies (Stevens, 1968), generate inappropriate procedures for psychometric and statistical analysis. Reliability coefficients and factor results are sensitive to monotonic (order-pre-serving) transformations of the variables, and thus results generated using linear psychometric models tend to be arbitrary, reflecting the arbitrary measurement scale selected for various variables. Monotonicity analysis represents a psychometric technique for determining the reliability and internal structure of variables which generates results that are meaningful and invariant under any linear or monotonic transformation of variables [11].

Whatever may be the varied nature of criticism of intelligence it is safe to say, that intelligence, when studied through psychometric methods, has provided us substantial information about its general contour or its general determinants. Nothing is considered sufficient in science until the problem is fully solved [1, 4]. And this was not considered sufficient by those who looked at the problem a bit more obliquely. They, therefore, saw the main problem of intelligence from the standpoint of development or growth. For doing this, they considered it very essential to observe the whole phenomenon carefully and cautiously with a view to classify and analyse it qualitatively rather than quantitatively. The study of intelligence then totally became a qualitative affair. Focus of study then lays on the manner of emergence of the various operations with age. With this mode of approach, successful search then could be made for any developmental trends, if really existing.

Piaget regarded development as continuous 'not only within the individual but throughout all evolutionary levels' [12]. He preserved the unity of nature in his theory which stretched from biological through social to intellectual levels, thereby explaining whith one clear stroke that the same fundamental processes functioned both in his beloved lowliest mollusk and the highly esteemed man of science of this century, that is, Albert Einstein [12]. Specifically speaking, Piaget saw intelligence as an extension of the biological adaptation. He, therefore, traced the growth of only those concepts which depended little on school instruction. Consequently, his investigation of intelligence took an unusual corner. Firstly, he disregarded the use of statistics. Instead, he preferred the use of symbolic logic especially designed for this purpose. Secondly, he knowingly did not use the various steps of scientific methods usually followed in traditional research in his treatment of problems. He, therefore, developed his own method now called the *Methode Clinique*. To make confusion worse confounded, he used different subjects for different tasks at various ages to report his results. Despite this lack of experimental control, he obtained results which were first considered of skeptic value, later on, turned out to be right, largely speaking, for his main aim was to find out the general mechanisms of intelligence and cognitive functions from their very earliest stages. Thirdly, he broadened the concept of the 'individual subject' and replaced him by the 'Epistemic subject'. He was thus highly fascinated by the intra-individual rather than interindividual phenomenon in regard to the

developmental aspect of intelligence. He, therefore, concluded that the study of the individual was less useful, and instructive than the study of human mind. Fourthly, it is least surprising that it became very difficult to subject his classified data to advanced statistical analysis. Fifthly, those experimentalists, who did, after replicating his experiments, got very funny results, reason being the complete ignorance of his theoretical frame of reference which had inspired his experiments [13]. For example, Seigfried E. Engelmann accused Piaget in the open of proving nothing [11]. Several others attacked him for solving neither this nor that problem [13]. What was then his secret of revealing new, novel and exciting information about the inside development of intelligence? In reply to this sort of criticism, Piaget has aptly remarked:

> Englemann said that I have proved nothing. He is quite right. There are many things that I have not proved. I think there are two ways of proving something: the first is to study one problem in great details as possible, using statistical methods, calculations of variations, and whatever else you may think feasible; the second is to keep moving from problem to problem, from field to field, seeking—and this is what counts—convergences and links between one field and another. Having for many years studied logico-mathematical operations with Barbel Inhelder, we have recently moved on to other problems: mental images, memory, processes of spontaneous learning, and so on. And for the last two or three years, I myself have concentrated on causality, where there remains so much to do and unravel. When you pass from one field to another, either there is a chaos or you find results which link up with observations already made. that is, you find all sorts of convergences and analogies. I personaly think it is far more satisfactory, as far as proof is concerned, to find these convergences and connections between fields than it is to work on only one problem using increasingly accurate statistical methods. Such statistical methods are of great value for critical purposes but it is not with them that we should start off [14].

It is quite clear from above that no basic contradiction is involved when intelligence is studied psychometrically or developmentally. This is precisely what Piaget did, while investigating intelligence developmentally and perception psychometrically, using rigorous controls in the latter case. In the former area, he perhaps loved being confused when he moved from problem to problem through a series of chains of confusions leading to clarifications again ending in confusions and so on. In this, the essence of what he talked was lost. M. Belanger, while reviewing research on

'learning studies in science education' restores this perspective succinctly when he says:

> Piaget is concerned with describing the "generalised knower" (The epistemic subject), and while such description sheds profound insight on epistemological problems, it lacks the fine structure required to be truly helpful in detailed curricular specifications and prescriptions. It seems that American researchers, who have particular talents in the techniques of empirics, could make a significant contribution to Piaget studies by investigating the fine structure and reporting what happens in detail within the stage of concrete operations between seven and twelve years of age for very specific science tasks [15].

Several specific studies are necessary to throw some light on the Piagetian system and over the years; it is hoped that cumulatively they may finally succeed to evaluate its relevance in its entirety.

Choice of Methods

After having discussed the relevance of factor analysis and the comparative advantages that accrue from investigating intelligence developmentally rather than psychometrically, it is necessary to say a few words about the choice of methods. It is safe to say that choice of method(s) should be relative to the problem under investigation for it is not an easy task to pigeonhole experimental procedures for all sorts of problems. This is why a combination of methods or procedures comes handy to tackle the problem effectively. One of the several methods available for investigative thinking, narrowly speaking, concept formation in children, is the Interview-Questionnaire Method. W. Edger Vinacke is of the opinion that this method, though full of pitfalls, is quite effective for procuring information through verbal inquiry, about the child's interpretation of objects and relations or about his understanding of natural, moral, causal, and other phenomenon' [16]. It is quite possible to question the child objectively provided care is taken to objectify the questions, manner of handling data, avoiding isolated whimsical responses, standardising the problematic situation, making the experimental procedure flexible enough to uncover the underlying quality of thinking demanded by a given problem, establishing rapport with the subject and being sensitive enough to possible errors in the interpretation of errors [16]. These demands made on the tool are quite exacting. Piaget struggled with this technique, now called the *Methode Clinique* for 'establishing the method of construction of true novelties' [11].

David Elkind comments on this *Methode Clinique* and says that it is nothing but a "combination of mental test and clinical interview—procedure which consists in the use of a standardised situation as a starting point for flexible interrogation" [16]. It is not an easy job to quantify this method for, according to Read D. Tuddenham, the "psychometric considerations must necessarily alter the format of cognitive problems originally approached by the *Methode Clinique*" [5]. This criticism in regard to the psychometrisation of the *Methode Clinique* is largely speaking, justified but, still, Piaget deserves our admiration for selecting and developing these very unconventional techniques in the face of several personal and professional odds for investigating human thought processes. This sort of unpalatable criticism is to be taken in its own stride of being in the business of research for it should not be lost sight of that every creative act ultimately rests on historical judgement as well as every scientific advance wheels on dogmatism, dynamism and discovery at the same time [4].

A Brief Resume

This background is necessary to understand the general orientation of this study as mentioned in the beginning of this chapter. When seen against this backdrop, it won't be inappropriate to point out at this stage that it has not been possible to employ *Methode Clinique* in this study due to the physical limitations of time as well as availability of subjects for intensive questioning on several diverse problems each containing a continuous chain of reasoning. Despite these handicaps, effort has still been made to obtain maximum intellectual performance of each and every pupil by administering problems individually in two sessions with generous time limits. It is in this context alone that the present study was designed to answer research questions of this study developmentally rather than psychometrically, attempting, in this process, to test a few of the hypotheses through factor analysis as mentioned in the beginning of this chapter. The design of the study is now discussed in the next section.

SECTION B: THE RESEARCH DESIGN

Aims and Objectives of the Study

The main aims and objectives of this study are:

(a) To study thinking (problem solving) processes, evoked by individual problems, containing a continuous chain of reasoning.

(b) To study the same processes when appropriately grouped, regardless of the typology of problems.

(c) To study errors as they occur in solving these problems.

(d) To determine the relationships between scores on thinking and some outside variables like: intelligence, sex, various immediate test reactions to the problems on presentation and adjustment.

(e) To find out the characteristics of successful and unsuccessful problem solvers.

(f) To analyse the structure of the appropriately grouped processes of thought factorially and interpret them psychologically.

(g) And lastly, to point out the main educational implications based upon the findings of this study for science teaching.

Hypotheses Proposed to be Tested

It is proposed to test the following hypotheses:

Ho_1: Problem solving takes place in stages.

Ho_2: The scores on problem solving are significantly related with the following independent variables: intelligence, sex, immediate test reactions to the problems on presentation and adjustment.

Ho_3: The complex problem solving processes arise from simple problem solving process.

Ho_4: A given problem is solved over a wide I.Q. range.

Ho_5: Poor problem solvers are influenced more by the content rather than the from of the problem.

Ho_6: There are significant differences in respect of the variables included in this study between successful and unsuccessful problem solvers.

Method of Procedure

In the preceding section, we have already made references to the general orientation of this study. To summarise, these related to process and a suitably modified interview questionnaire in contrast to product, psychometric and objectives methods of procedure. Further, the use of statistical techniques was subordinated to this

end. This step facilitated collection of data as well as its objective scoring. Edgar Vinacke has supported the use of this approach for it is equally respectable with other approaches used in the investigation of concept formation, attainment and problem solving in retaining the various thinking processes in their contextual characters [1,16]. To conclude, the entire procedure comprised the following steps:

(i) Sample.
(ii) Features of the problems chosen.
(iii) Developing the schemes of thought.
(iv) Other tools used.
(v) Use of statistical devices for the analysis of data.
(vi) Limitations of the study.

(i) *Sample*

It is a field experiment, one out of the four categories of social scientific research, in which a non-probability sample of 200 pupils—100 boys and 100 girls each—ranging in age from 10.5-11.5 years to 14.5-15.5 years corresponding to grades VI through X or length of schooling in complete years ranging from five to nine was selected on the basis of their intelligence and socio-economic status. The following constraints were further imposed on this sample.

Firstly, the names of all the pupils belonging to grades VI to X were written on a piece of paper along with their dates of birth. The senior students supplied their own dates of birth. For others, the reliance was placed on the school register. On request, the class teachers of the various classes were obliging enough to fill in the socio-economic status information card as developed by Dr. Kuppuswamy [17]. This card was scored by the investigator. All those students who belonged to the lower middle socio-economic status and were within the particular age group as mentioned above but of the same grade were selected and the rest were eliminated from the study straightaway. This step incidentally resulted in a restricted sample for intelligence testing as well.

Secondly, the Group test of Mental Ability as developed by Dr. S.S. Jalota was administered to the above sample [18]. The individual raw scores and ages being known, the corresponding mental ages were read from the table. All the Intelligence Quotients were thus computed and then arranged in the ascending order both gradewise and sexwise from 70 I.Q. points upwards. Thirdly, it was attempted to obtain 20 suitable I.Q. points on the linear dimension of intelligence ranging from 70 to 120. These twenty I.Q. points

were: 70, 72, 75, 77, 80, 82, 85, 90, 92, 95, 97, 100, 102, 105, 107, 110, 112, 115, 117 and 120. Fourthly, attempt was made to select as many students as possible from the same school with the restriction that all the pupils finally included in this sample came only from those schools which offered the same course, drew their pupils from more or less the same socio-economic status and were under the same administrative control (state government in our case). The sample was finally drawn from five higher secondary schools, two boys and three girls. In short, the sample finally selected had the following characteristics:

(a) The grade was controlled objectively, and in turn, age within each grade was also restricted. Incidentally, this also eliminated failure cases from grade to grade.

(b) All the pupils from less than average socio-economic status as measured by Dr. Kuppuswamy's scale were included in this sample.

(c) All the pupils had their I.Q.'s ranging from 70 to 120. Within this range, twenty I.Q. points were chosen which facilitated matching from grade to grade.

(d) All the individual subjects of the sample were selected finally in consultation with the concerned class teachers, who knew them fairly well. Here, the class teachers relied heavily on current and past achievements of the pupils, keeping in mind the fact that each subgroup represented controlled intelligence (by the investigator) and the achievement fairly well.

(e) Within each age group, the boys and girls were also matched on intelligence.

(ii) *Features of the problems chosen*

At the very beginning of this investigation, it was rightly felt that the success of this study would depend upon the individual characteristics of the investigative problems, finally selected for this study, the manner of administration of problem also being no exception. The development of the test instrument has been described in depth in the next section. Here, it will only suffice to state the major features of the diverse problems finally selected for the study. These were:

(a) The problems, when considered individually, should be as diverse as possible.

(b) They should be new, novel and interesting to science students. At the same time, they should provoke thought and demand variety in thinking.

(c) They should be general, that is, they should not be too bookish or demand any specialised knowledge in their solution for the study intended to investigate thinking processes over a wide age range.

(d) The reading difficulty of all the problems should be intentionally kept low. This will ensure the comprehension of the individual problems.

(e) Every pupil should be able to solve part of the problem. For if a problem is too easy, it is likely to be attempted successfully by all. If it is too difficult, it is not likely to be attempted by any one and hence of trivial significance in this study.

(f) As far as possible, problems, when considered individually, could be broken up into thinking processes (or problem solving processes) whose mastery by the individual pupils could be judged or ascertained through questioning them objectively.

(g) The number of problems should not be too large but at the same time, all the problems included in this study, when considered cumulatively should adequately sample thinking relevant to the learning of science. To concretise, the problems, when robbed of their typology would attempt to measure certain aspects of thinking, particularly speaking, when thought processes are appropriately grouped. These were called schemes of thought in this study.

(iii) *Developing the schemes of thought*

It was considered necessary to analyse individual problems at depth in terms of thinking (problem solving) processes which could be, later on, regrouped in schemes of thought on logical basis. It is also necessary to consider the genesis of this term as used in psychological literature as well as the meaning given in this study with special reference to science. In British psychology, R.C. Oldfield and O.L. Zangwill analysed the meaning of this term in their paper on 'Head's Concept of Schema' and its application in contemporary British Psychology about thirty years ago. Head suggested this term while studying 'control of bodily movement and localisation of tactile stimulation [19]. For example, a man is able to scratch his left or right ear from several diverse starting positions. He, therefore, hypothesised the existence of Image which a man carried in his head which exactly corresponded with its physical counterparts, a case of kinaesthetic perception. F.C. Bartlett, while giving similar treatment to memory, took his cue from response and stated that skilled behaviour is essentially schematic in character [19]. To put in other words, it meant that scheme not only determines any

part of the response but also the whole sequence of responses which could be flexible under certain conditions. Piaget gave this term a comprehensive meaning and rooted it in biological behaviour (scheme).

Let us consider scheme first. It is the same thing as schemas, plural being schemata. If loosely considered and understood, it could cover several terms like 'strategy, concept, plan, decision process, orientation, sequence of links, structure of hierarchical nature, learning set, cognitive structure, determining tendency, set of equivalences, search model or strategy, awareness of total lay-out, utilisation of logical rules, and key factor to a varying degree each having past, present and future' [19]. To strengthen this consideration further, a schema at any moment can be considered to be composed of sub-schema, each sub-schema then being also called schema with mutual implications of varying interconnectedness at that very moment of development. At the time of its origin, it encompasses strategy and, in general, reflects relations among strategies at every level. So it is easier to comprehend that a schema comprises strategies or sub-schemata, a few of them, at least, active during problem solving. It has the inherent potentiality to organise itself at different levels. If not available at a particular moment, an inferior one may become active and seek accommodation to the problematic situation fully.

It is then not difficult to see that the various schemata, as they develop, are related to each other with the inherent possibility of the superior one controlling their inferior allies. Only when seen thus, a psychological organ parallel to physiological one is not only contemplated but also created on a very simple and plausible principle: all behaviour is related or essentially schematic in character without even suffering any loss of individual contextual meanings of the various schemata. So schema is nothing but the sort of algebraic generalisations or even individualised procedures of the available strategies taken individually in groups having content and form which disciplines the inside. If this workable view of schema is accepted, then it is not difficult to state the following attributes or functions of schemata as propounded by Piaget:

(a) It is as simple as reflective activities and as complex as advanced problem solving plan. It thus grows and organises itself at different levels, both horizontal and vertical.

(b) It is invisible, that is, not open to direct observation. It is, therefore, inferred only.

(c) It assimilates as well as accommodates to external reality.

Over the years, it is raised to the concept of a psychological organ having its parallel physiological counterpart.

(d) It does not develop in a vacuum. It is also not empty from inside. It is active as well as flexible having both content and form. The latter disciplines the former, for example, differentiating. In summary, the internal structure and the differentiated functions among the various components of the scheme are mutually inter-related, even compensation being no exception.

(e) In case of failure, another schema may become active for it is in the nature of schema to act on a situation: physical or mental. This implies the existence of possible relationship among schemata. The very discrepant situation causes strain in the organism, which meets it by the very growth of schema sooner or later. Otherwise, it shows its coordinating functions 'with greater possibilities for the transfer of generalised search strategies' [19].

(f) Lastly, it reflects itself by a given behaviour sequence or in it. the very utilisation of logical rules which are comparatively speaking, easy to determine from the very knowledge which is being investigated upon from the angle of psychological formation. It is, thus, easy to see that the "schema becomes the set of equivalences implied by the behaviour of the subject—not, of course, in any single sequence, for no equivalence could be inferred from any isolated observation, but in any adequate set of related observations" [19].

It is, therefore, least surprising that their psychological investigation is a very exciting problem which has to be seen against two additional basic standpoints of the Geneva school. The first is that complex thinking processes arise or stem from simple thinking processes. Secondly, intellectual development (or thinking) takes place in stages for which Piaget has listed five criteria [20]. This adds its own difficulties to the psychometric treatment of Piaget's concepts.

Let us now refer to the scheme of thought logically derived at in this study. First, in studying adolescent thinking, we are more concerned with the processes of thought than with the products of thought. Our interest, then, is the way the various answers to a series of test items in a particular problem are reached. As thinking processes, unlike physical objects, are not open to direct observation and the various techniques have their own respective advantages and disadvantages, one can only hope to know certain key points which map the route right from the presentation of the problem to the moment the problem is finally solved. It was, therefore, considered

desirable to analyse these problems in terms of thinking processes needed to solve all the problems regardless of their type. Secondly, another problem remained: How to define a thinking (problem solving) process? This was solved by developing the following criteria for deriving thinking processes from the various problems:

(a) It is not simply an addition or subtraction process applied mechanically. On the other hand, it is backed up by reasoning.

(b) Within the context of each problem, mastery over it and others in turn is likely to lead to the solution of the problem.

(c) It is fundamental in character. In this light, each test item has been examined to determine whether it did constitute a thinking (problem solving) process.

(d) And lastly, the number of thinking processes should not be unnecessarily too large.

For carrying out this sort of analysis, it became very necessary to know how the solutions to the various problems grew. All the seventeen problems were given a preliminary run and the various thinking processes evoked were listed. These were again scrutinised closely by a small group of ten method specialists [21]. The following twelve schemes of thought were, therefore, contemplated in the first instance.

(i) Using constant difference.
(ii) Using summation.
(iii) Using proportion.
(iv) & (v) Combinatorial grouping: Beaker combinations and digital combinations.
(vi) Generalisation to algebraic symbols.
(vii) Stating hypotheses.
(viii) Testing hypotheses.
(ix) Stating procedures.
(x) Formulating problematic situations.
(xi) Using insight.
(xii) Failure to grasp the essence of the problem.

During the main study and especially, at the tabulation stage of the various thinking processes, it appeared that the three schemes of thought could be improved upon by reclassifying the thinking processes included in them. For example, the combinatorial problem on four digits when solved fully gave sixty different responses which could be reclassified into: using two digits at a time, using three digits at a time and using four digits at a time. The generalisation to algebraic symbols earlier considered as one variable

broke into two, that is, generalisation to algebraic symbols (summation); and generalisation to algebraic symbols (proportion). The problem on formulating problematic situations on cow and cycle also split into two: fluency (number of problems proposed) and flexibility (typology of problems which could be loosely equated with flexibility). Thus the total number of schemes increased to seventeen which are mentioned below along with the serial number of their corresponding contact problems as well as the thinking processes. Number of processes of thought and maximum score for each scheme of thought are also shown in the table given below:

Table 2.1. Scheme of thought, serial number of contact problems as well as process, number of processes and the maximum score possible on the various schemes of thought

S. No.	Scheme of thought	S. No. of contact problems	S. No. of processes	No. of processes	Maximum score
1	2	3	4	5	6
1.	Using constant differance	1, 2, 3, 6, 15	2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 41, 101b	18	19
2.	Using summation	5, 6, 7, 14, 15	33, 34, 36, 48, 83, 85, 90, 92, 95, 96, 97	11	11
3.	Using proportion	4, 5, 6, 7, 14, 15	28, 29, 30, 31, 32, 37, 39, 40, 46, 47, 80, 82, 87, 93, 98	15	16
4.	Beaker Combinations	13	74, 75, 76, 77	4	14
5.	Using two digits at a time	9	50	1	12
6.	Using three digits	9	51	1	24
7.	Using four digits	9	52	1	24
8.	Generalisation to algebraic symbols (summation)	2, 3	13, 14, 21, 22	4	8
9.	Generalisation to algebraic symbols (proportion)	4	26, 27	2	4
10.	Stating hypotheses	17	103	1	Open
11.	Testing hypotheses	17	104, 105, 106,	3	6
12.	Stating procedures	14, 15, 7	94, 101a, 107	3	6
13.	Proposing tests	16	102	1	Open

1	2	3	4	5	6
14.	Formulating problematic situations (fluency)	12	72	2	Open
15.	Formulating Problematic situations (flexibility)	12	73		Open
16.	Insight	8, 11	49, 66, 67, 68, 69, 70, 71	7	20
17.	Failure to grasp the essence of the problem	10	54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65	12	17

It is necessary to make a remark at this stage. These seventeen schemes of thought should not at all be regarded as having been used either in the Piagetian context or a bit elaborated in the historical context as pointed out by E.A. Lunzer [22]. Here, their very consideration has been from a very limited angle, that is, that these schemes of thought do appear to play their part in learning science, especially physical sciences. (At their best, they can be equated with scientific skills, a usage which has not been so far considered in literature.) Nobody can deny that science students do not make computations involving constant differences (in relation to a given reference), using summation (adding and subtracting), using proportion (multiplying and dividing), generalising to algebraic/symbols (using algebraic symbols according to certain set criteria) and combinatorial groupings (combining or classifying according to certain set rules). Moreover, science students have also to demonstrate certain other behaviours. Examples are: stating hypotheses, testing hypotheses, proposing tests, formulating problematic situations (both fluency and flexibility), using insight and grasping the essence of the problem. This was the only justification for hypothesising the existence of these schemes of thought. In short, at this stage, they are only hypothetical constructs requiring confirmation through factorial analysis.

(iv) *Other tools used*

In addition to the test instrument especially designed for this study and described in the next section, only three other tools, whose descriptions are avilable easily were used. These were:

(a) The Group Mental Ability Test (1960) by S.S. Jalota [18].

(b) Socio-economic Status Scale by Kuppuswamy [17].

(c) Personality Inventory on Adjustment by M.S.L. Saxena [23].

(v) *Use of statistical devices for the analysis of Data*

Being a qualitative study, it is first proposed to discuss each problem separately, that is, how it is solved by the pupils under study in relation to the demands of the given problem. As each problem, largely speaking, inheres a continuous chain of reasoning, only key points in it will be tested. Errors made by the pupils during problem solving will also be noted, studied and their frequencies computed. Such data will be presented gradewise as well.

Secondly, in continuation, each problem solving process within the problems included in this study will be examined separately and its mastery, both within and across the five age groups, will be judged. Within the context of the flexible questioning for every problem, it is also proposed to ascertain the presence or absence of the individual thinking processes, under study, thus, diminishing to a great extent the incidence of errors made by pupils due to chance.

Thirdly, the various individual thinking processes, regardless of the typology of problems, will be grouped on the principle that all those processes which appeared to belong together will be placed or bracketed together. These appropriately grouped processes of thought will be called schemes of thought. Data were so presented as to answer the following two questions: (a) Which of the processes are difficult to acquire? (b) Do the complex problem solving processes arise from simple problem solving processes?

(vi) *Limitations of the study*

Since the study was experimental in nature and the tool was to be administered individually, the following were the main limitations of this study:

(a) The sample selected for this study was limited to 200 urban subjects, girls and boys, equal in number, ranging from grades VI to X drawn from the same socio-economic status.

(b) The study is strictly limited to the students of science especially during the last two years at school. Other subjects had studied general science up to grade eighth.

(c) The present study does not aim at constructing a standardised test for measuring the various processes of science for it studies thinking as a developmental phenomenon. So no attempt is made to psychometrise the instrument as well as to establish any predictive relationships among variables included in this study.

(d) Only the processes of thought relevant to problems, of course, diverse in character, were traced. Other aspects of thinking were excluded. Examples are: functional fixedness, thematic thinking, creative thinking as popularly understood and anagrams, that is, breaking the code.

(e) No hints and cues, except within the very statement of the problem, were given. For example, giving the meaning of a difficult word or giving clarification on a certain word or phrase or problem solving phase within the context of the problem itself.

(f) No specific content in science was covered as the problems were investigated developmentally. Otherwise, it would have been difficult to avoid the use of specialised vocabulary and even concepts. At the same time, they retained their scientific flavour to a considerable extent. Thought so evoked could be analysed in three different ways: problem, process and scheme of thought wise.

(g) The various processes of thought underlying the seventeen problems included in this study were first analysed logically and, later on, classified and grouped intuitively regardless of the typology of the problems, on the principle that all those processes which appeared together should be grouped together separately. To diminish the bias ot the investigator, only the opinions of ten method specialists were taken.

(h) Whereas a few problems were demonstrated as well as students were allowed to handle equipment, they were not required to collect evidence as is usually done in the science laboratory during experimentation. However, they were put in a situation where each of them could reason out things for himself.

SECTION C: DEVELOPING THE TEST INSTRUMENT

Locating the Problems

First, a few problems each containing continuous chain of reasoning were discussed with IV year B.Sc., B.Ed., students, B.Ed., science students and M.Ed., science students of the Regional College of Education, Ajmer in the 1972-73 session. In turn, they were asked to think out other problems of similar nature which not only tested thinking in several ways but also contained little specialised scientific content. It was done because these problems were to be designed for those who were studying in several grades. Some of the problems suggested by the trainees of the Regional College of Edu-

cation, Ajmer were also discussed in the classroom. To this list, other problems arising out of varied discussions were also added as well (N = 45).

Screening the Problems

The screening of the various problems collected above underwent three stages. Firstly, ten senior physics and mathematics teachers were requested to judge the suitability of these problems on the basis of their own experience with the adolescent pupils [21]. Altogether, they rejected 15 problems. Secondly, ten method specialists were requested to score the remaining thirty problems on a three point scale. All problems with a mean of less than 2 were dropped [24]. This reduced the number of problems from 30 to 22 [25]. Thirdly, the writer himself administered all these 22 problems in two sessions on three students each drawn from grades VI to X with a view to familiarise himself with the type of thought processes evoked by these problems. Here, the main considerations underlying this preliminary run were: determining the time taken to solve each problem, absence of opportunity to solve the problem experimentally, inclusion of difficult or technical words, and extent of thought provoked by the problems. This preliminary run of the problems indicated that three problems were little solved by grade X children and the other two problems were solved fairly well by pupils studying in grades VII and VIII. Consequently, seventeen problems were left which were attempted in varying degrees by these pupils in three to four hours in two sessions.

In the light of this experience, these seventeen problems were rewritten for final administration. It is of passing interest to mention that the majority of these pupils showed considerable interest while tackling these problems. Quite a few among them not only continued to solve these problems after the school bell but also reported their solutions to the investigator even 3-4 days after the test experiment was administered. This in a way reflected the suitability of these problems for inclusion in the study. A letter which contained instructions for the benefit of pupils was also drafted. In summary, the following seventeen problems were included in this study:

(i) Height Problem
(ii) Positive Constant Difference Problem
(iii) Negative Constant Difference Problem
(iv) Proportion Problem
(v) Hotel Problem
(vi) Rectangle Problem

(vii) Rectangular Cubes Problem
(viii) Counting Maximally Rectangles Problem
(ix) Combinatorial (Digital Problem)
(x) Questions Inviting Wrong Answers Problem
(xi) Nine Dots Problem
(xii) Formulating Questions Problem
(xiii) Beakers Problem
(xiv) Fish Problem
(xv) Spring Balance Problem
(xvi) Proposing Tests Problem
(xvii) Flow of Water Through a Tube Problem

Manner of Administering the Problems

It has already been pointed out that one of the main aims kept in mind was to obtain maximum intellectual performance on each and every problem of the test instrument. For this reason alone, problems were administered individually in two sessions. Restrictedly speaking, individual mode of administration in contrast to case study: *Methode Clinique* only ensured that the problems would command not only their firm attention but also, by the very nature of the experimental setting, they would care to attempt the entire test to the best of their abilities. There was another advantage in this approach as well. At the end of each problem, the key points in their various thinking processes could be tested with a view to judge their mastery on them through a series of questions separately mentioned under each problem. In order to avoid excessive amount of writing on the part of pupils, the test instrument was printed. This meant in other words that a pupil would write either in the space provided or anywhere on the page or he would do the problem in any way he liked best. If found necessary (or requested), extra sheets were also given. Lastly, informal atmosphere was maintained throughout the experiment by giving the meanings of difficult words, if any, and providing a few procedural clarifications which by their very nature did not supply answers to the various test items of the problems The testing time ranged from two hours and a half to three hours and a half per pupil.

Recording of Data During Administration of Problems

It is a double pronged study: problemwise and scheme of thought-wise. The scores on the latter were derived from the scores on problems where each problem had its own scoring key. Naturally, special data sheets for the collection of data were designed, which,

largely speaking, followed the scheme of scoring. This served two purposes. Firstly, a score could be obtained for any individual on any problem. Secondly, a score could also be obtained not only for any one of the problem solving processes within any problem but also across the various problems and consequently, problem solving processes. The latter were grouped into seventeen schemes of thought as already mentioned in the preceding pages. Moreover, errors committed during problem solving were recorded on these sheets. Other events of psychological interest were also noted. Data thus became available for 28 variables which are now mentioned below:

Tests	*No. of variables*
(i) Intelligence	1
(ii) Grade	1
(iii) Adjustment: home, health, social, emotional & school	5
(iv) Immediate test reactions to the problems on presentation: understanding, felt difficulty, confidence and interest in solving problems	4
(v) Number of problems or Number of schemes of thought	17

It may be added that while analysing the structure of the problems factorially, both the problems and the schemes of thought were pitted against each other with a view to facilitate the studying of internal structure of problems, thus, raising the total number of variables to 45.

Features of the Test Instrument

Let us now pinpoint the main features of the test instrument. These are:

(a) It contains seventeen thought provoking problems, diverse in character, thus, inviting variety of thought, judged suitable for investigation by method specialists and practising teachers. This goes a long way in enhancing its face as well as content validity.

(b) When the individual process of thought regardless of the typology of problems are reclassified on the principle of similarity and belongingness, it again contains seventeen schemes of thought (developed empirically at the end of the study). If true, their strengths and weaknesses can be judged effectively.

(c) It touches but not embraces the *Methode Clinique*. Still it is administered individually. Unlike it, it does not expect individual pupils to handle the equipment firmly. Moreover, it also lacks its sophistication.

(d) In continuation, at the same time, it is quite flexible in its mode of administration in the sense that it, strictly speaking, allows for formulating and reformulating of the same problem within its meaning with a view to make a given problem fully comprehensible to the individual pupils.

(e) It is a face to face instrument administered in two sessions in which the experimenter can try hard to set at ease any individual pupil.

(f) It also collects data on immediate test reactions to some of the problems on presentation which are scored on a three point rating scale. Four summated scores, so available, can be used as outside variables for statistical purposes for they are real and can be relied upon like the letter grade system of examination.

(g) It is process-based rather than content-based for problems neither require any specialised knowledge nor are they based on any specialised branch of science. They are too general and hence logical in character. They, however, still retain their scientific flavour.

(h) It is a power rather than a speeded test instrument, there being no rigid time limits for the solution of the various problems in two sessions. Three to four hours in two sessions are more than generous for any individual.

(i) It is a printed instrument so it reduces amount of writing. It is also possible to obtain oral responses if so desired.

(j) It stimulates rather than inhibits pupil responses for it neither causes fatigue nor boredom when administered in two sessions.

(k) Its other psychometric aspects, that is, reliability and validity, of course, not essential in a study of this type are reasonably high (see below).

(l) It is least based on memory.

Reliability and Validity of the Test Instrument

It is always desirable to have information about the reliability and validity of the test instrument. When the information was sought from some of the experts in the field, the opinion was found to be divided [26]. Whereas R.P. Bhatnagar considered the whole exercise unnecessary, E. A. Lunzer suggested determining the intercorrelations among the various piagetian tasks as well as the use of test-retest method for determining the reliability of the test. W.J. Jacobson

emphasised the importance of having some faith in regard to reliability and validity of the test instrument and then added, "It is not the aim of this study to psychometrise the test instrument quite a big task in its own right" [27]. B.K. Passi and Amarjit Singh confirmed Jacobson [28]. The choice lay either in choosing blunt problems for investigation accompanied by significant loss of information on individual thinking processes or using a test of questionable reliability and validity. There was another difficulty as well, that is, whether the scores on diverse problems, a few of them quite open, could be added. Allied to this, for which grade are these two coefficients to be reported? It was after all decided to compute these coefficients, first, for every scheme of thought and secondly, for grade IX in which most of the pupils begin to solve these problems quite substantially. Test-retest method was used for determining the reliability coefficients of the various schemes of thought. For determining validity coefficients, the following external criteria were chosen: (i) Intelligence as measured by Jalota's test. (ii) Two class teachers' combined rating on problem solving ability of pupils.

The reliability and validity coefficients of the test instrument in terms of schemes of thought are now reported below:

Table 2.2. Test-retest reliability as well as validity coefficients of the seventeen schemes of thought for grade IX pupils

S. No.	Scheme of thought	Reliability coefficient	Validity (intelligence)	Coefficients problem solving ability
1	2	3	4	5
1.	Using constant difference	.89	.83	.60
2.	Using summation	.91	.88	.37
3.	Using proportion	.86	.91	.67
4.	Beaker combination	.67	.64	.35
5.	Using two digits at a time	.96	.07	.24
6.	Using three digits at a time	.95	.96	.53
7.	Using four digits at a time	.98	.91	.51
8.	Generalisation to algebraic symbols (summation)	.88	.59	.50
9.	Generalisation to algebraic symbols (proportion)	1.00	.45	.22
10.	Stating hypotheses	.68	.86	.69
11	Testing hypotheses	1.00	.35	.08
12.	Stating procedures	Variable does not change		
13.	Proposing tests	.68	.73	.32

1	2	3	4	5
14.	Formulating problematic situations (fluency)	.91	.84	.40
15.	Formulating problematic situations (flexibility)	Variable does not change		
16.	Using insight	.90	.31	.11
17.	Failure to grasp the essence of the problem	.73	.53	.40

Except a few fluctuations, most of the reliability coefficients are within the acceptable limits. Regarding validity coefficients, one has to be quite careful in their interpretations. Intelligence as measured by current intelligence tests is an omnibus concept. It comprises several diverse individual abilities. Examples are 'perception, language skills and sensori motor coordination'. In contrast to this, Piaget regards intelligence as rational processes like 'reasoning, problem solving and concept formation'. It is therefore least surprising, according to W. Heyne Reese and Lewis P. Lipsitt, that 'correlations between performance on Piaget type tasks and performance on I.Q. tests are not exceptionally high' [29]. Taking an overall view, like reliability coefficients, most of the validity coefficients lie within the acceptable range as suggested by J.P. Guilford. To quote Guilford:

> ...expect reliability coefficients to be in the upper brackets of r values, usually .80 to .98; and validity coefficients to be in the lower brackets, usually .00 to .80 [30].

Limitations of Test Instrument

Any test instrument of this type which investigator thinking processes during adolescence possesses certain inherent limitations. These are:

(a) It does not reflect achievement in science in any of its specific branches.

(b) Its testing time is on the high side as well as a pupil has to come twice to the researcher for the test instrument is administered in two sessions.

(c) It only tests key thinking processes at key points leaving, in the process, something to the speculation of the researcher. It is particularly so because thinking processes are not open to direct observation.

(d) It ignores the role of hints and cues in investigating human thinking. This restricts to a varying extent the performance of potential problem solvers.

(e) It is not a standardised instrument as understood in statistical sense. Conventional item analysis has not been carried out either on individual problems or the various thinking processes because that would have resulted both in blunt problems for investigation as well as loss of significant information on the various processes of thought as the succeeding chapters on the description of individuals will show.

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20. Bolton Neil. The Psychology of Thinking, Ibid. pp. 50-52.

21. List of Science Education Method Specialists who were contacted.

The following were contacted (* They only responded):

*(i) Shri J. K. Sood, Reader in Education, Regional College of Education, Ajmer.

*(ii) Shri S.B. Singh, Regional College of Education, Ajmer.

*(iii) Shri S.N.L. Bhargava, Regional College of Education, Bhopal.

*(iv) Dr. J.S. Grewal, Regional College of Education, Bhopal.

(v) Shri M. K. Gupta, Field Adviser, Regional College of Education, Bhopal.

*(vi) Shri S.P. Sharma, Regional College of Education, Bhopal.

*(vii) Dr. J.S. Rajput, Regional College of Education, Bhopal.

*(viii) Dr. A.B. Pathak, Teachers Training College, Udaipur, Rajasthan.

*(ix) Shri Pritam Singh, Department of Textbooks, N.I.E., N.C.E.R.T., New Delhi.

(x) Shri Ved Prakash Rattan, Department of Science Education, N.I.E., N.C.E.R.T. New Delhi.

(xi) Dr. R.C. Sharma, Department of Textbooks, N.I.E., N.C.E.R.T., New Delhi.

(xii) Dr. V.K. Kohli, Vice-Principal, Khalsa College of Education, Amritsar (Punjab).

*(xiii) Shri B.K. Sharma, College of Education, Kalakankar, U.P.

*(xiv) Shri S.M. Aggarwal, University College of Education, Simla, H.P.

(xv) Dr. J.N. Joshi, Department of Education, Punjab University, Sector 14, Chandigarh.

22. Lunzer E.A. The Regulation of Behaviour, Ibid.

23. (i) Saxena M.S.L. Manual for Vyaktitva—Parakh—Prashnavali (M.A. 62), Deptt. of Psychology, B.H.U., Varanasi-5. pp. 1-8.

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24. List of problems other than finally accepted.

S. No.	Problem considered	Overall rating
1.	Numerical analogies: 100 - 200 200 - 300 300 - 400 500 - 600 600 - ? ? - ? X - ? ? - Y	2.4

S.No.	Problem considered	Overall rating
2.	100 × 1 = 100 100 × 2 = 200 100 × 3 = 300 100 × 4 = 400 ——— —— ——— —— ——— —— 100 × 10 = ? 100 × ? = 1500 100 × ? = 2000 X × ? = 3000	2.3
3.*	A train goes from Delhi to Ajmer via Jaipur. The fare from Delhi to Ajmer is Rs. 30/-. If fare from Ajmer to Jaipur is Rs. 10/-, what will be the train fare from Delhi to Jaipur?	2.4
4.*	In how many possible ways can you think of identifying the given pole of an unmarked magnet?	2.6
5.	Frame as many questions as you can on flowers whose answers you do not know.	1.9
6.	You are given a barometer. In how many possible ways can you use it to measure the height of your school?	1.5
7.	By how many possible methods can you measure the velocity of wind?	1.8
8.	Ingenhauz Apparatus.	1.9
9.*	Germination of seeds	2.8
10.	Lever experiment	1.5
11.	Forbis experiment	1.2
12.*	Simple pendulum	2.9
13.	Questions involving wrong answers: (i) A person with one eye closed can count two birds on a tree. How many birds will be count with two eyes on the same tree? (ii) Two men carry each a bale of cotton and a bale. Who carries more load?	2.6

*Eliminated during the pre-administration.

S.No.	Problem considered	Overall rating
	(iii) What is this circle? (iv) Which is more heavier: two kilos of cotton or two kilos of iron? (v) How many donkeys should be killed to obtain the following number of legs: (a) 10 legs (b) 13 legs (c) 23 legs	

25. Kelly E. Lowell. Assessment or Human Characteristics, Prentice Hall of India, Pr. Ltd., New Delhi, 1960. pp. 35-50.

26. Professor R.P. Bhatnagar. Dean, Faculty of Education, Institute of Advanced Study, Meerut University, Meerut, U.P.

27. Professor W.J. Jacobson, Chairman, Deptt. of Science Education, Teachers College, Columbia University, Columbia, New York.

28. (i) Dr. B.K. Passi, Professor of Indore University, Indore.

(ii) Dr. Amarjit Singh, Lecturer cum Research Officer, School of Education, University of Reading, Reading.

29. Reese Hayne W. and Lipsitt Lewis P. Experimental Child Psychology, Academic Press, New York and London, 1970.

30. Guilford J.P. Fundamental Statistics in Psychology & Education, McGraw Hill Book Company Inc., New York etc. Kogakusma Company Ltd., Tokyo, Asian Students' Edition, Third Edition, p. 146.

CHAPTER III

THE INDIVIDUAL DESCRIPTION OF PROBLEMS
(Qualitative Analysis)

Structuring the Presentation

In this chapter, it is proposed to describe qualitatively the various thinking processes evoked by the seventeen problems, problem-wise among certain groups of adolescent pupils, both boys and girls (N=200). There is only one exception, that is, problems 2, 3 and 4 have been discussed consolidatedly for they involve an interesting construct when processes underlying them are aggregated under the title: 'problems based upon numerical analogies'. Incidentally, this saves the space as well. Secondly, only a few problems of interest are detailed in terms of its elements and aims, manner of presentation, scoring, sample responses, errors committed, distribution of dominant errors gradewise and summary of results. Thirdly, depending upon individual problems and availability of commonable treatment, little deviation has been made, while presenting data on the acquisition of the various individual processes of thought within the thought structure of a given problem under study both within and across the various sub-samples. Fourthly, statistical relationships and overall conceptual interpretations find their place in the succeeding chapters. Lastly, it is necessary to make the following remarks in general in regard to the presentation of data on selected problems so that any reader is guided effectively while going through the data as well as arguments built around the research aims or hypotheses set up in this study. To reiterate even at the cost of repetition, these few points to be kept in mind while reading all these problems are:

1. The study population is 200 pupils, 100 boys and 100 girls. They have been drawn in equal numbers from five grades, namely,

grade VI, VII, VIII, IX and X. In each grade, there are 20 boys and 20 girls.

2. In each grade, 20 I.Q. points have been selected which range from 70 to 120. These I.Q. points are 70, 72, 75, 77, 80, 82, 85, 90, 92, 95, 97, 100, 102, 105, 107, 110, 112, 115, 117 and 120. There is a pupil on each one of these I.Q. points within all the various sub-samples gradewise as well as sexwise.

3. All the pupils, sexwise, have been arranged in the ascending order of I.Q. Therefore, boys and girls have been denoted as follows:

I.Q.	S. No. of Boys	S. No. of Girls	Grades
70	B-1, B-21, B-41, B-61, B-81	G-1, G-21, G-41, G-61, G-81	VI, VII, VIII, IX, X
72	B-2, B-22, B-42, B-62, B-82	G-2, G-22, G-42, G-62, G-82	VI, VII, VIII, IX, X
120	B-20, B-40, B-60, B-80, B-100	G-20, G-40, G-60 G-80, G-100	VI. VII, VIII, IX, X

These scripts appear when sample responses are discussed. Thus pupils sex, grade and I Q. are fixed straightaway.

4. Data, generally speaking, have been presented in the following tabular forms. For illustration:

(i) Showing means and standard deviations gradewise as well as sexwise for the various sub-samples: VI (boys and girls); VII (boys and girls); VIII (boys and girls); IX (boys and girls) and X (boys and girls).

(ii) Showing the number of pupils having a particular score (0, 1, 2 or 3 etc.) gradewise as well as sexwise and range of I.Q. for the various sub-samples. High frequency in the cell means that the problem is failed or passed over a wide I.Q. range. For example, if the frequency in the cell is 5, this means that the range of I.Q. is minimally 5 points (which has very rarely appeared consecutively on the I.Q. Scale).

(iii) Showing the extent of acquisition, in terms of number of pupils, of the various individual thought processes gradewise as well as sexwise for the various sub-samples. Thus are calculated the facility values for the pooled sample.

(iv) Showing errors committed, number of errors committed and the distribution of only dominant error(s) gradewise for the various

sub-samples. On few processes of thinking, even two to three dominant errors have also appeared. The grade distribution for other errors has not been shown because their number is too small to discern any pattern, largely speaking.

5. Data have been presented in such a manner that qualitative findings appear, quite frequently, on direct inspection of the table. In these cases, it has been given straightaway with direct reference to the concerned table. This has diminished overcrowding of data within the cell as well as the table, the latter being successively derived from the original presentation of data.

6. Inferences have been drawn within individual sub-samples as well as across sub-samples. Within individual sub-sample, sex has been another variable which bi-classifies each sub-sample. In other words, it means:

(i) For the pooled sample, each pupil carries a percentage of .5 (N=200).

(ii) Within each grade, each pupil carries a percentage of 2.5 (N=40).

(iii) Within each grade, each boy and girl carries a percentage of 5 (N=20).

Depending upon the context, it is possible to compute percentages from frequencies of pupils and, in turn, frequencies of pupils from the percentages of pupils.

7. Only correlations of interest between the problem under study and the remaining 44 variables and other observations or events of interest have been given in the summary of results at the end of the problem.

8. Hump effect has been hinted at when the incidence of given error increases with age. An error is said to be dominant when it is shared by more than 15 per cent of the pupils.

9. Experimenter's remarks or questions appear in the text as usual. They have been bracketed only or written a bit away from the text when confusion in reading is suspected.

10. The symbol 'R' against any one of the thinking processes means that it relates to the reading aspect of the problem. Or alternatively, the answer to the test item based on this (R) process is contained within the body of the problem itself. All such processes have not been scored.

11. Only a few problems of significant interest have been discussed at depth due to limited space available. In regard to others, only statements of problems, manners of presentation and scoring and main findings have been given.

Lastly, the serial number of problems, hypothesised thinking processes and schemes of thought remain the same throughout their treatment.

1. HEIGHT PROBLEM

The Problem

Ram in taller than Shyam by 2 cms. Ram is taller than Mohan by 6 cms. Ram is shorter than Sohan by 6 cms. If Ram's height is 200 cms, then answer the following questions:

Question asked	Process No.	Score
1. What is the height of Ram?	1	R
2. What is the height of Mohan?	2	1
3. What is the height of Sohan?	3	1
4. What is the height of Shyam?	4	1
5. How much is Mohan shorter than Shyam?	5	1
6. State the procedure (eliminated from scoring for those who solved this problem, successfully could also verbalise the procedure)	6	R
	Maximum score	4 marks

Manner of Presentation

The problem was presented as stated. Pupils were asked to read and reread the problem while hesitant as well as while justifying their reasons. No other hints were given.

Elements and Aims of This Problem

It is an interesting problem because it deals with the determination of individual heights of three children involving the use of constant difference. It is as well a hypothetical problem because data supplied are fictitious. In its simplest form, it comes very close to the experimental problem on conservation of length in which two equal sticks are placed side by side with ends coinciding exactly (and then disturbed slightly lengthwise) as used by Piaget while investigating the thought processes of pre-operational children [1]. If length is replaced by colour, this problem stands in close comparison to the problem used by Burt while investigating formal reasoning among adolescents: "Jane is fairer than Lily; Jane is darker than Susan;

which of the three is fairest" [2]? This problem is normally attempted by the adolsecent pupils when it is presented concretely. In physics, several situations arise when physics teacher tries hard to put across concepts involving the use of constant difference, for example, use of zero error in the use of vernier callipers or conversion of temperature from centigrade scale to the fahrenheit scale and *vice versa*. Some of these considerations led to the choice of this problem. This problem, when presented as mentioned above, aims at investigating the following:

(a) Up to what extent can the adolescent pupils structure the problem successfully?

(b) Up to what extent can they handle the arithmetical operations relating to shorter than, taller than, and by how much, when the reference height is given?

(c) Up to what extent can they verbalise their methods of attack?

Presentation of Data

Table 3.1.1. Number of pupils gradewise as well as sexwise experiencing difficulty in reading the problem and range of I.Q. for the various sub-samples

		VI	VII	VIII	IX	X
	Boys	20	3	5	—	—
	Girls	7	15	1	1	1
I.Q.	Boys	70-120	70-120	80-120	—	—
Range	Girls	70- 90	70-117	102	92	70

Table 3.1.2. Means and standard deviations gradewise as well as sexwise for the various sub-samples

S. No.	Grades	Sex	Mean	S.D.
1.	VI	Boys	1.5	2.14
		Girls	1.4	1.91
		Boys+Girls	1.45	3.52
2.	VII	Boys	.80	1.29
		Girls	.25	.94
		Boys+Girls	.53	2.28
3.	VIII	Boys	1.9	1.81
		Girls	2.6	1.91
		Boys+Girls	2.25	3.79
4.	IX	Boys	3.6	1.02
		Girls	2.65	1.85
		Boys+Girls	3.13	3.14
5.	X	Boys	3 05	.92
		Girls	3.35	1.54
		Boys+Girls	3.20	2.62

Table 3.1.3. Number of pupils having a particular score gradewise as well as sexwise and range of I.Q. shown in brackets for the various sub-samples

Grade	Sex	0	1	2	3	4
VI	Boys	8 (70-110)	5 (72-107)	—	3 (82-117)	4 (97-120)
	Girls	13 (70-117)	—	—	—	7 (92-120)
VII	Boys	13 (72-120)	2 (102-110)	3 (70-105)	—	2 (97-107)
	Girls	18 (70-117)	1 (115)	—		1 (120)
VIII	Boys	8 (70-107)	2 (80-112)	2 (90-92)	—	8 (85-120)
	Girls	7 (72-112)	—	—	—	13 (70-120)
IX	Boys	1 (92)	—	2 (77-95)	—	17 (70-120)
	Girls	6 (70-107)	1 (90)	—	—	13 (77-120)
X	Boys	—	1 (92)	5 (70-85)	6 (75-105)	8 (90-120)
	Girls	1 (75)	1 (85)	—	2 (82-102)	16 (70-120)

Table 3.1.4. Acquisition of thought processes in terms of number of pupils gradewise as well as sexwise for the various sub-samples

		Processes						
Grade	Sex	2	3	4	5	6		
						0	1	2
VI	Boys	11	7	7	4	14	4	2
	Girls	7	7	7	7	13	—	7
VII	Boys	5	4	4	2	18	—	2
	Girls	1	1	2	1	19	—	1
VIII	Boys	12	9	8	8	12	—	8
	Girls	13	13	13	13	7	—	13
IX	Boys	19	19	17	17	3	—	17
	Girls	13	13	14	13	7	—	13
X	Boys	19	19	15	8	3	9	8
	Girls	18	17	19	17	2	2	16
	Total	118	109	106	90	98	15	87

Summary of Results

Statistics, when presented in tables, indicate the following inferences:

It is necessary to make two observations. In grade VI, all boys felt difficulty in reading the problem for not a single pupil among them could correctly give the height of Ram (Process No. 1 which was not scored because answer to the test item was contained in the problem itself). In the same grade, only 35 per cent of the girls experienced this sort of difficulty. Secondly, there is a rise and fall in this percentage with a time lag of one year in both sexes. For example, in grade VII, the percentage of girls rose up to 75 per cent whereas in case of boys of the same grade the percentage fell down to 15 per cent. In the next immediate grade, the latter registered increase and the former decrease in percentage of those who were so affected. In grades IX and X, the reading part of the problem was well mastered by pupils of those grades. The reason for this appears to be that this problem by its very nature baffles pupils and each one is affected differently over limited grade levels but, at the same time, over a wide I.Q. range as shown by 3.1.1.

Let us now consider average performance on this problem gradewise as well as sexwise. First, there is dip in performance in grade VII both for boys and girls. Secondly, mean performance favours boys in grades VI, VII and IX and girls in grades VIII and X. It appears boys and girls try hard to equalise their mean performance as they move up the grades. Thirdly, as expected, average performance increases consistently with increasing grades except in grade VII.

It is a matter to be further examined why slightly more than half the pupils of grades VI and 77.5 per cent of pupils of grade VII failed to solve this problem. The percentage declined to 37.5 in case of grade VIII pupils, sexwise percentages being 40 (boys) and 35 (girls). It is only in grades IX and X that the problem begins to be solved a bit satisfactorily, the gradewise percentages being 75 and 60 respectively. The same percentage is retained when it comes to the verbalising of methods of attacking the problem. There is another interesting observation as well. As judged by frequencies in the various cells, a given problem is solved (or failed) over a wide I.Q. range not only within individual grades but also across the various grades (see Table 3.1.3).

Except a dip in grade VII, pupils show gradual mastery increasingly over the various processes when considered in isolation, from grade to grade. Secondly, it is only from grade VIII onwards that pupils are in a position to verbalise their methods of attack as

shown by the table below (see the acquisition of process number **6** in Table 3.1.4).

	VI	VII	VIII	IX	X
Boys	30%	10%	40%	85%	85%
Girls	35%	5%	65%	65%	90%

Thirdly, the large number of frequencies in cells both within and across the various grades show that a given process is as well mastered over a wide I.Q. range. Fourthly, considering the pooled sample, the various processes were mastered as follows (see Table 3.1.4).

Process No. 2	=	59	per cent
Process No. 3	=	54.5	per cent
Process No. 4	=	53	per cent
Process No. 5	=	45	per cent

On its face value, the problem appears to be very simple, which it is not, when considered gradewise. It would have been possible for our pupils to solve this problem successfully had they cared to draw three lines of unequal lengths representing the heights of Ram, Mohan and Shyam. No one among them did this simple exercise; and consequently, poor performance resulted on the last process (see the acquisition on process number 5 in Table 3.1.4).

Sample Responses

Let us mention below a few sample responses on the various processes of thought. Two general questions of the experimenter were: 1. How did you find this or that answer? 2. How did you approach this problem?

1. G 4, I.Q. 77, Grade VI, Score 0.

Ram's height, you know, is 200 cm. It is given. Mohan's height is 104 cm because there are two boys (100 cm each). So Mohan's height is 100+(6-2) cm. Shyam's height is 196 cm (200-4). Hence Mohan is taller than Shyam by 4 cm because (200-196). He appears to be quite confused first due to names, and then due to figures followed by phrases: 'taller than', 'shorter than' and 'who is shorter than'? He attempts item to item making use of any available figure on the way. Why four only? Because the difference can be only that much and not more! So answer is correct but reasoned out wrongly.

2. G 10, I.Q. 85, Grade VI, Score 0.

Yes, the height of Ram is 200 cm. The height of Mohan is 4 cm because, you see, 6−2=4 cm. You see, you also get 4 when you

subtract Shyam's height from the height of Sohan. There is something wrong with the question because the height of Ram is large but the heights of Mohan, Shyam and Sohan are too small. This happens in mathematics because after all I am either to add or subtract. Hence, this is the right answer according to my way of thinking.

3. G 69, I.Q. 92, Grade IX, Score 0.

The height of Ram is 198 cm. Why? Because $200-2=198$ cm. It shows confusion between Ram and Shyam. The height of Mohan is 4 cm because $6-2=4$ cm. The height of Sohan is 0 cm because $6-6=0$. The height of Shyam is 0 cm because $4-4=0$. I do not think I can do this question, Sir.

4. B 39, I.Q. 117, Grade VI, Score 0.

The height of Ram is 200 cm. How? You can see it. The height of Mohan is 800 cm. because $6+2=8$ metres $(8\times100=800$ cm). No! His height is 820 because $800+2\times10=820$. The height of Sohan is 860 cm because $820+4\times10=860$ cm. The height of Sohan is 900 because $860+4\times10=900$ cm. The little Mohan (difference between the height of the two) is 100 cm because $900-800=100$ cm. The problem is solved, using this argument straightaway step by step.

5. G 82, I.Q. 72, Grade X, Score 4.

The height of Ram is 200 cm. The height of Mohan is 104 cm because $\frac{200}{2\text{ (boys)}}+4\ [\because\ 6-2\ ]$. The same is the height of Sohan in my opinion. Why? Let me think! The height of Ram is 200 cm. Now the height of Shyam is 198 cm $(200-2)$. The height of Mohan is 194 because $200-6$ cm (Mohan is shorter). The height of Sohan is 206 cm because $200+6$ (Sohan is taller or Ram is taller); and Mohan is shorter than Shyam by 4 cm or I can also say that Shyam is taller than Mohan by 4 cm It is the same thing, you know. Earlier I found the average and was wrong.

6. B 100, I.Q. 120, Grade X, Score 4.

The height of Ram is 200 cm because it is given. The height of Shyam is 198 cm because he is shorter than Ram by 2 cm. The height of Sohan is 206 cm because he is taller than Ram by 6 cm or also Ram is shorter than Sohan by 6 cm. Similarly, the height of Mohan is 194 cm because he is shorter than Ram by 6 cm. We know now that the heights of Mohan and Shyam are 194 and 198 cm respectively. So Shyam is taller than Mohan or Mohan is shorter than Shyam by the same amount, that is, 4 cm.

The Growth of Solution

Let us see how the solution of this problem grew. It appears to have undergone the following stages. First, it is necessary to distinguish among the three names. Ram, Mohan and Shyam. Why? Because calculations made on any one of the referrents can be switched to each other for obtaining the right answer. Secondly, the phrases: 'smaller than' or 'taller than' baffled many, the main victims being the pupils of grade VII for 65 per cent boys and 90 per cent of girls of this grade could not solve this problem at all. Quite a few among them thought of cracking this problem at process No. 4, which involved finding the height of Shyam. The success on this item gave them a bit of confidence to solve the problem again from the first process to the last process.

In their responses, one sees another line of attack. The height of Ram is 200 cm and the number of boys left after excluding Ram is two: Shyam and Mohan. Therefore, the average height is 100 cm. [$\because \frac{200}{2}$]. The other supporting figures are 2 cms and 6 cms which relate to phrases 'smaller than' and 'shorter than'. Those are then operated on the average figure that is, 100 cm. If this does not work, then operate on those figures in isolation with little concern for the reference height of Ram (200 cm) as given in the problem. On reflection, this can't be true because then the friends of Ram turn out to be very very small or pygmies. It is, therefore, preferable to return to the figure 200 cm on which again plus and minus operations, are made but with little concern for names. If this does not work then the height of Mohan is 8 metres, because metre is much much larger than cm. In this context, the height of Mohan is 8 metres. Why? Because $6+2=8$ metres. 2 and 6 as already mentioned, are the supporting numbers which provide key to the solution of the problem. They could well be multiple of 100 when cm in their thinking changes to decimetre. This shows that pupils float variety of ideas while solving a given problem without examining them at depth for which they have their own set of reasons based upon past experience.

Analysis of Errors

While solving this problem, pupils committed several errors. Except a couple of cases, all of them gave reasons for their answers and, in the process, did seek clarifications which were given only within the statement of the problem. (For example, the height of Ram is 200 cm., yes, it is.). Pupils of grades VI and VII read the

problem carelessly and began to fill in the various answer spaces with the assistance of available figures 2, 6, 6 and 200. To illustrate, not a single pupil in grade VI could supply the height of Ram straightaway on reading the problem in the first instance because they least expected the answer within the problem itself. It is on reading and re-reading the problem that they found that the problem under solution was trickier than the usual ones which they traditionally encountered in their day-to-day classroom teaching. At some moments, they showed lapses of attention, for example, 200−2=98 which was rectified later on. There were other moments, of course, quite a few in number, where they made double errors. For example, =200—2=98=196=198. The table given below mentions errors committed on the four processes of thought. Also are shown dominant errors (italicised) along with their grade distribution.

Table 3.1.5. Errors committed as well as number of errors committed along with the distribution of only dominant error(s) grade-wise for the various sub-samples

S. No.	Process No.	Errors committed	No. of errors committed	Dominant error	Grade-wise	No. of pupils
1.	2	*64*, 104, 2, 3, 6, 600,	21	*6*	VI =	16
		800, 196, 206, 298,			VII =	21
		820, 106, 202, 94,			VIII =	11
		1996, 118, 194,			IX =	3
		101, 198, 114.			X =	—
2.	3	*6*, 198, 200, 2, 94,	20	*6*	VI =	16
		8, 14, 600, 800,			VII =	23
		194, 4, 0, 10, 292,			VIII =	10
		12, 860, 446, 2008,			IX =	4
		106, 192			X =	—
3.	4	*2*, 200, 6, 4, 108,	18	*2*	VI =	8
		192, 7, 202, 5, 286,			VII =	14
		900, 98, 41, 196,			VIII =	5
		198, 1, 12, 206			IX =	3
					X =	—
4.	5	*4*, 6, 15, 2, 600,	18	*4*	VI =	5
		199, 194, 60, 8,			VII =	8
		200, 202, 100, 10,			VIII =	12
		12, 208, 105, 198, 1			IX =	5
					X =	1

Discussion on Errors

From Table 3.1.5 the following findings emerge:

(a) Pupils have committed all sorts of errors on very simple test items. The number of errors made on the four individual processes of thinking are 21, 20, 18 and 18. This number is quite large and unexpected which shows abundant individual differences in thinking on pinpointed questions. In other words, this also shows that each pupil sees the problem in his own characteristic way.

(b) Dominant errors have an overall tendency to decrease with grade which is an expected finding. For example, errors completely disappear in grade X except a single frequency in grade X on process No. 5. But before they decline, they suffer a hump. Once this is crossed, errors disappear. A hump is shown below for the dominant error committed on process No. 2 where this error is shared by 51 pupils, i.e. 25.5 per cent of the sample.

(c) There is abundant variety in their answers for which they appear to have strong reasons. It is not difficult to interpret dominant errors for they are the supporting numbers to be seen in relation to the height of Ram. In the absence of the latter, all other errors have appeared whose frequencies are too small to deserve a special treatment. However, it is necessary to make a remark here, that is, these errors have appeared, disappeared, reappeared, and persisted from grade to grade but in small numbers.

(d) Lastly, this problem correlates positively and significantly at one per cent level more with grade than with intelligence, the correlations being .484 and .341 respectively. It correlates negatively and significantly with the felt difficulty of the problem. It correlates insignificantly with social adjustment, confidence in the problem, interest in the problem and formulating problematic situation (flexibility). With other problems and schemes of thought, it correlates positively and significantly.

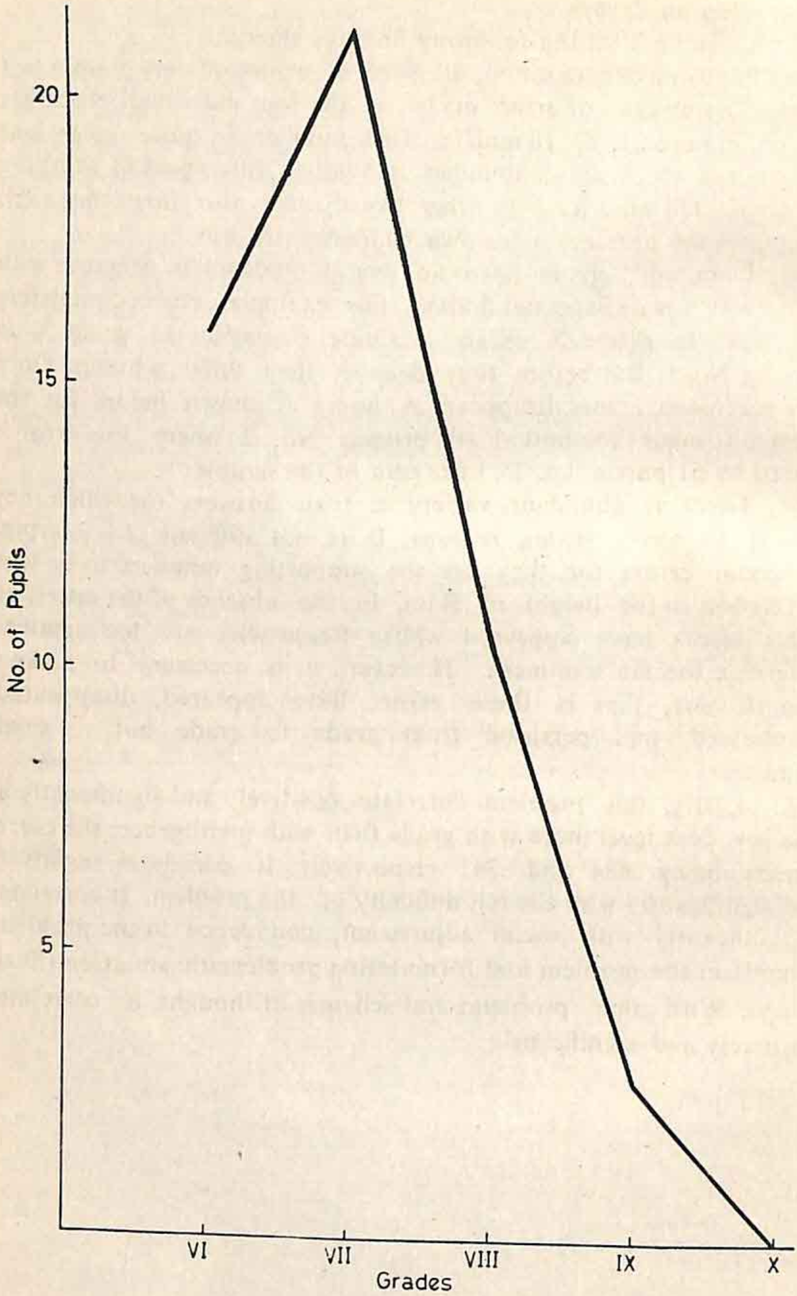


Fig. 2. Hump of dominant error on process number 2 of problem no. 1.

2, 3 AND 4: PROBLEMS BASED UPON NUMERICAL ANALOGIES

Problem 2—Positive Constant Difference Problem

We have written some figures against each other in two columns. As you go down the columns, there appears some relationship. Try to understand it. Then fill in the blanks indicated by the question marks (?).

Test items		Process No.	Score
0	5	—	—
5	10	—	—
10	15	—	—
15	20	—	—
20	25	—	—
25	?	7	1
?	35	8	1
?	40	9	1
40	?	10	1
?	50	11	1
50	?	12	1
Continue			
X	?	13	2
?	Y	14	2
		Maximum score	10 marks

Manner of Presentation

Problem was presented as stated above. Attention was, however, drawn to the various question-marks on the sheet. No other hint was given. The remaining two problems were also presented in the same manner.

Maximum score = 10 marks

Problem 3—Negative Constant Difference Problem

Read the figures in the two columns carefully. There is some relationship in these figures as you go down the two columns simultaneously. Fill in the spaces indicated by the question marks.

Test items		Process No.	Score
—5	—10	—	—
—10	—15	—	—
—15	—20	—	—
—20	—25	—	—
?	—30	15	1
—30	?	16	1
Continue			
—45	?	17	1
?	—55	18	1
?	?	19,20	1, 1
Continue			
—X	?	21	2
?	—Y	22	2
		Maximum score	10 marks

Problem 4—Proportion Problem

Read the figures opposite to each other given in the two columns. Try to find the relationship. Fill in the spaces by appropriate figures.

Test items	Process No.	Score
10 × 1 = 10	—	—
10 × 2 = 20	—	—
10 × 3 = 30	—	—
10 × 4 = 40	—	—
10 × ? = 50	23	1
10 × ? = 70	24	1
10 × ? = 100	25	1
Continue		
10 × ? = X	26	2
? × 10 = Y	27	2
	Maximum score	7 marks

Elements and Aims of the Problems

Like the first problem, these three problems on reading appear to be simple, a bit more concrete and deal with arithmetical numbers, both positive and negative, as well as algebraic symbols involving constant difference and proportion. The superficial impression of their being simple, however, is deceptive for they require

forming concepts on the part of pupils a, they read down the columns. They are then to fill in the blank spaces (question mark spaces); and then to generalise the same to the algebraic symbols placed on both sides of the sign of equality (=). While doing so, they had to give reasons to their various answers. As expected, pupils did not experience any reading difficulty on these problems except that most of the pupils, particularly, of grades VI through IX missed the various question marks (spaces) which were shown. These three problems, when taken together, aimed at investigating the extent to which pupils can advance their reasoning on the basis of their formed concepts. The problems, when presented in the form already stated, aimed at finding the following:

(a) Up to what extent can they form the concept of positive constant difference based on arithmetical data?

(b) Up to what extent can they form the concept of negative constant difference based upon similar type of data?

(c) Up to what extent can they extend or generalise their reasoning to the algebraic symbols involving constant difference and proportion?

Presentation of Data

Table 3. (2, 3, 4).1. Means and standard deviations gradewise as well as sexwise for the various sub-samples of problems 2, 3 and 4.

Grade	Sex	Means of problem			Standard deviations		
		2	3	4	2	3	4
VI	Boys	5	.5	3	2.13	1.40	—
	Girls	2.4	.8	3	1.91	1.83	—
	Boys & Girls	3.7	.65	3	3.52	3.27	—
VII	Boys	5.9	1.9	3	.44	2.71	—
	Girls	4.5	.7	3	2.60	1.71	—
	Boys & Girls	5.2	1.3	3	3.98	4.17	—
VIII	Boys	5.4	3.3	3	1.80	2.98	—
	Girls	6.0	1.8	3	0.00	2.75	—
	Boys & Girls	5.7	2.25	3	2.62	5.93	—
IX	Boys	6.7	5.4	3.8	1.31	3.01	1.60
	Girls	5.7	5.1	3.2	1.31	2.14	.87
	Boys & Girls	6.2	5.25	3.5	2.80	5.28	2.65
X	Boys	6.1	6.2	6	.44	.87	1.73
	Girls	6.1	6.9	5.4	1.61	1.61	1.96
	Boys & Girls	6.1	6.55	5.7	2.35	3.75	3.75

Sample Responses on Generalisation to Algebraic Symbols

One finds several interesting responses made on algebraic symbols. Processes of thinking at work here are somewhat as follows:

(i) You see that the figures are going down or up by 5 so I can go up like this up to 70-75. But there is no end to it for I have to solve the question.

So X 75 ——— 0 There has to be relationship here.
0 ——— Y

I am confused. Then writes

X 75
and
? Y

If you so wish, put X=0 and Y also. This is done in algebra, you know.

(ii) I am to think on these symbols. Let me give you the figures: 60, 8, 55, etc. But these do not help me.

(iii) My answer is Y because X and Y go together. If neither X and Y, it is W because the other way round W and X go together. If neither, then X Y go together, and hence responses like XY, YX, YZ, ZY.

(iv) I got one step up, so my answer is X+1, and hence consequent answers like X+1 or Y+1. If you are not satisfied, answer has to be larger; and hence double it.

(v) There is another observation. My answer is X=Y+5 (How did you obtain it ?) because Y has to go somewhere. Two questions are combined, in which two distinct processes of thought, are involved, namely process No. 13 and 14. Even when the context of the question changes, the underlying reasoning of those who struggled with these symbols remained the same.

To reiterate, these algebraic symbols had no sense for most o our pupils. With a view to handle these symbols successfully, they thought of concretising these symbols, at least, clearly on five out of the six thinking processes. Largely, pupils have committed large number of different types of errors while handling any given symbol, their frequencies being 12, 16, 20, 18, 11 and 11 respectively on these processes. One is really impressed by their variety in thinking when any single group of errors is considered or looked at. One gets excited when any experimentor encounters response of this type on process No. 13:

Yes, I see that numbers are rising up by 5. It is straight up to

50—55

I can fill in the other numbers which are

55--60
60—65
65—70
70—75 and so on.

There is no limit. When it comes to X, I am really flunked. It could be

	X	5
	5	
	X	5
5	X	X
	5	
5	X	X
	5	

I cannot think of writing it in any other way.

(What should it mean any way?)

This is what I am thinking.

Stops thinking.

Then suddenly says:

It could be Y or W.

(Which means following the alphabet and reversing it around X).

Summary of Results

Let us now summarise the main results on these three problems.

Pupils did not experience any reading difficulty on these problems except that majority of pupils of grades VI to IX missed quite a few question marks or answer spaces as they solved these problems.

As expected, average performance increases with grade. Considered sexwise, and gradewise, mean performance favours boys rather than girls. But the gain favouring boys is marginal because as they move into higher grades, both sexes try hard to equalise their performance.

A given problem is solved or failed over a wide I.Q. range within each grade as well as across the various grades. As judged by large frequencies in the various cells, this equally applies to various thinking processes evoked by the three problems. In continuation, there is a general tendency among the pupils to acquire mastery over the various thinking processes.

The second problem involving positive constant difference correlates positively and significantly more with grade than with intelligence, in fact, the correlation with the latter being insignificant (r=.0611). The same is true in the case of the fourth problem, its correlation with intelligence being r=.0409. This, however, is

not true of the third problem involving negative constant difference, the corresponding correlation of this problem with intelligence being positive and significant (r=.2044). The reason for this appears to be that most of the pupils have performed excellently on the arithmetical items of these two problems, bringing about little variation in test scores. Except few fluctuations in case of the second problem, there is a general tendency among the three problems to correlate positively and significantly with all the remaining problems as well as all the schemes of thought. All the three problems (except a single fluctuation) are insignificantly correlated with the various immediate test reactions to the problems on presentation.

5: HOTEL PROBLEM

The Problem

Ten persons from a village went to a hotel for an evening meal. They ate 4 *chapatis* per head at the rate of 10 paise per chapati; 10 plates of vegetables at the rate of 50 paise per plate; one cup of ice cream each, costing a rupee per cup. After meals each one of them drank a cup of coffee costing 50 paise each. At the end of their meals, they gave two ten rupees notes to the manager of the hotel. Now answer the following questions.

Questions asked	Process No.
1. How much in all did they spend on *chapatis*?	(28)
2. How much in all did they spend on vegetable plates?	(29)
3. How much in all did they spend on ice cream?	(30)
4. How much in all did they spend on coffee?	(31)
5. How much did they spend per head?	(32)
6. How much did they spend in all?	(33)
7. How much additional money was demanded by the hotel manager?	(34)
Maximum score	7 marks

Manner of Presentation and Scoring

The problem was presented as written above. Pupils were allowed to read and re-read the problem as many times they wished. Each process carried one mark.

Elements and Aims of the Problem

It was considered desirable to add a concrete problem to the list of problems under consideration not only for the sake of variety but also it was assumed that a concrete problem involving money with which children have had some experience would evoke sufficient thought relating to three schemes of thought: Using constant difference, summation and proportion. Secondly, it was suggested that it be rephrased in such a manner as to demand at least a careful reading of the problem on pupil's part. With this end in view, the problem aimed at finding out how the various processes were mastered by the pupils from grade to grade. This problem is, therefore, aimed at investigating the following:

(a) Up to what extent could pupils calculate money spent on individual items in different context?

(b) Up to what extent could they compute the total expenditure as well as per head expenditure?

(c) Up to what extent could they compute correctly the additional money yet to be paid on behalf of the manager?

Presentation of Data

Table 3.5.1. Means and standard deviations gradewise as well as sexwise for the various sub-samples

S. No.	Grade	Sex	Mean	Standard deviation
1.	VI	Boys	3.5	2.17
		Girls	4.35	1.98
		Boys & Girls	3.93	4.23
2.	VII	Boys	5.1	3.62
		Girls	3.1	2.72
		Boys & Girls	4.1	6.71
3.	VIII	Boys	5.7	3.65
		Girls	3.85	2.01
		Boys & Girls	4.78	6.17
4.	IX	Boys	6.8	.88
		Girls	5.49	2.27
		Boys & Girls	6.15	3.68
5.	X	Boys	7.0	0.00
		Girls	5.85	1.80
		Boys & Girls	6.43	2.76

Summary of Results

The main results on this problem indicate:

1. As expected, mean performance increases with grade. Except in grade VI, boys score higher than girls in all the remaining grades. The problem is fully solved by 10th grade boys.

2. All pupils except 15 per cent from grade VI, 22.5 per cent from grade VII and a girl each from grade VIII and IX have had a try on this problem. If judged by the large frequencies of pupils having a particular score, it is safe to conclude that a given problem or part of the problem solved over a wide I.Q. range not only within each grade but also across the various grades.

3. Except few fluctuations here and there, there is a general tendency among pupils to show increasing mastery over the various processes as they move into higher grades. Girls try hard to equalise their performance with boys in grade X.

4. Let us now see their failure on the seven processes of thinking underlying this problem.

(i) It is strange to note that expenditure on *chapatis* involving the number of men, *chapatis* and cost per *chapati* could not be computed correctly by over one fifth of the pupils from grade VI to VIII, the individual corresponding gradewise percentages being: 27.5, 35, 22.5, 15 and 2.5.

(ii) Comparatively simple items like the total expenditure on vegetables, ice cream and coffee have attracted a good percentage of pupils from the lower grades who could not make simple straight forward calculations, their gradewise percentages are given below:

Table 3.5.2. Percentages of pupils failing on the seven processes of thought gradewise

S. No.	Processes	Grades				
		VI	VII	VIII	IX	X
1.	Expenditure on *chapatis*	27.5%	35%	22.5%	15%	2.5%
2.	Expenditure on vegetables	25%	27.2%	7.5%	5%	5%
3.	Expenditure on ice cream	37.5%	32.5%	10%	5%	2.5%
4.	Expenditure on coffee	37.5%	35%	7.5%	5%	2.5%
5.	Total expenditure per head	70%	52.5%	42.5%	20%	20%
6.	Total expenditure	65%	52.5%	32.5%	13.5%	12.5%
7.	Money demanded by manager	72%	55%	37.5%	20%	12.5%

It is, therefore, least surprising that when it came to the computations of expenditure per head, total expenditure and money demanded by the manager, the failure rate among the lower grades shot up. There is, however, one promising feature. There is a general decline in failure rate from grade to grade. But the performance of the pupils judged by large percentages in grade VI and VII is still not satisfactory on a simple straight forward problem (involving money) included in this study.

5. The problem correlates positively and significantly both with grade and intelligence, the correlations with grade being more than with intelligence ($r=.4491$; and .2482). It correlates negatively and significantly with confidence in the problem ($r=-.1526$). It correlates insignificantly with home adjustment, understanding the problem, felt difficulty of the problem, flow of water problem (testing hypotheses), stating hypotheses and formulating problematic situations (fluency). With all other problems and schemes of thought, it correlates positively and significantly.

6: RECTANGLE PROBLEM

The Problem

There is a rectangle which is 25 cm long and 15 cm wide. A man goes four times around it at the rate of 4 cms per second. Now answer the following questions.

	Questions asked	Process No.
1.	Draw a diagram	35 R
2.	Show the distance on it	35 R
3.	What is the total distance when the man goes once round the rectangle?	36
4.	What is the total distance when the man goes four times round the rectangle?	37
5.	How much time does he take for covering a distance of 4 cms?	38 R
6.	How much time does he take for going once round the rectangle?	39
7.	Suppose he rests for a second at the end of each round. How much time does he take for completing just four rounds?	40, 41

Manner of Presentation

The problem was presented as written above. The pupils were allowed to read and re-read the problem if they so desired. Each process carried a mark each except the last one which carried two marks. Maximum score=6 marks.

Elements and Aims of the Problem

It is a very simple problem with which most of the pupils studying in a middle school are familiar. However, it has been presented a bit differently. The problem involves finding the perimeter of the rectangle, calculating time for going once round it at the rate of 4 cms per second; again the same for making four rounds; and lastly, taking into account separately or otherwise the time spent in rest at the end of each round. In this context, the problem does demand thinking for it cannot be done correctly mechanically. This problem, therefore, is aimed at investigating the following:

(a) Up to what extent can the pupils read this problem effectively?

(b) Up to what extent can they handle successfully the following calculations: determination of perimeter; time taken to complete one as well as four rounds; and the time spent in taking rest at the end of each round.

(c) What errors do they make while solving this problem?

Presentation of Data

Table 3.6.1. Means and standard deviations gradewise as well as sexwise for the various sub-samples

S. No.	Grade	Sex	Means	Standard deviation
1.	VI	Boys	.2	.87
		Girls	0.0	0.00
		Boys & Girls	.1	1.25
2.	VII	Boys	0.0	0.00
		Girls	0.0	0.00
		Boys & Girls	0.0	0.00
3.	VIII	Boys	.65	1.15
		Girls	.2	.87
		Boys & Girls	.425	2.04
4.	IX	Boys	3.1	1.18
		Girls	.6	1.43
		Boys & Girls	1.85	3.62
5.	X	Boys	2.00	2.00
		Girls	1.35	1.74
		Boys & Girls	1.68	3.80

Summary of Results

Let us now summarise the main findings on this problem. These are:

1. The performance on this problem has been, generally speaking, poor throughout the grades. Perhaps due to the calculations made mechanically, pupils have failed to analyse the various variables involved in this problem. Consequently, the grade means have not increased prominently with age. In grades IX and X, boys have performed better on this problem than girls despite their dip in performance in grade X.

2. Throughout the grades, the problem is failed over a wide I.Q. range. Except a single fluctuation in grade VI across the various processes, pupils of grades VI and VII have failed to show mastery on all the thinking processes. From grade VIII onwards, the various processes are gradually mastered increasingly from grade to grade.

3. Largely speaking, most of the pupils have approached the solution of this problem mechanically. For example, they have started on a wrong foot when they failed to distinguish between perimeter and area in relation to this problem despite the fact that they had earlier indicated the distinction correctly on the drawn rectangle itself. Consequently, the elementary test items have attracted a large number of errors which range from 9 to 36, each taken in isolation, which indicates that they accept every possible stake in their thinking in the hope of getting the right answer. For this, they make use of every possible phrase, figure or unit available within the statement of the question.

4. Errors decline with increasing grades but before they do so, they undergo clear humps. When it comes to answering a test item whose answer is contained in the body of the problem itself, as many as 26 different errors appear, interestingly enough, dominant errors on it also undergo a hump.

5. It correlates positively and significantly more with grade than with intelligence, the two correlations being .425 and .335 respectively. It correlates positively and significantly with all the problems as well as the schemes of thought except positive summation series problem and generalisation to algebraic symbols (summation). It is significantly correlated with understanding of the problem felt difficulty of the problem, confidence in the problem and interest in the problem.

7: RECTANGULAR CUBES PROBLEM

The Problem

We have three rectangular glass cubes, each measuring 3 cms in length, 2 cms in width and 1 cm in thickness. They were put one by one in a large measuring jar. Before immersing them in, the level of water in the jar stood at 40 cm. Now answer the following questions.

	Questions	Process No.	Score
1.	What was the level of water before immersing the three rectangular cubes?	42	R
2.	What is the length of the rectangular cube?	43	R
3.	What is the width of the rectangular cube?	44	R
4.	What is the thickness of the rectangular cube?	45	R
5.	What is the volume of the rectangular cube?	46	1

It is given to you that a rectangular block of one cm length, one cm width and one cm thickness raises the level of water in the cylinder by 1 cc.

6.	What will be the level of water in the cylinder on immersing the three rectangular cubes?	47, 48	1 each

Maximum score = 3 marks

Manner of Presentation and Scoring

The graduated cylinder and the three cubes were shown. The problem was presented as stated above. Each process carried one mark.

Elements and Aims of the Problem

It is a very simple problem of application type which evokes seven processes in all out of which three were considered significant for scoring purposes, each carrying one mark. The latter processes involve finding the volume of the given cube ($3 \times 2 \times 1$); computing the volumes of three cubes (3×6); and using this value to determine the rise in the level of water. The latter figure is to be added to the original level of water, that is, 40 cms on the basis of the information already given in the question. This information was that a rectangular block of 1 cm length, 1 cm width and 1 cm thickness raises the level of water in the graduated cylinder by

1 cc. The basic consideration in the selection of this problem was to judge the extent to which the adolescent pupils could utilise the given information in solving this problem. In this context, the main aims and objectives of this problem were:

(a) Up to what extent can the adolescent pupils determine the volume of the cube?

(b) Up to what extent can they determine the volume of the three rectangular cubes?

(c) Up to what extent can they determine the final level of water reached on the basis of the (known) information already supplied in the body of the problem.

Reading Difficulty of the Problem

It has already been mentioned that graduated cylinder and three rectangular cubes were shown to every pupil. Pupils of grades VII, IX and X did not experience any difficulty in reading the problem. Only a single pupil (I.Q. 95) of grade VIII experienced difficulty in answering questions to the problem whose answers were already contained in the very problem itself. In Grade VI, on the other hand, 4 boys and 4 girls experienced difficulty in answering the above mentioned questions. The I.Q.'s ranged from 80 to 110 in the case of boys; and 77 to 112 in case of girls. Most of their difficulties lay in distinguishing between cm and cc (cubic cm); writing dimensions of the cube but in wrong spaces, using the figures of 40 or 6 (derived from 3×2) or 41 $(40+1)$; and if not cm then perhaps a kilometre. These pupils constituting one fifth of the sample with I.Q.'s ranging quite widely needed help in reading the problem, despite the fact that the main elements of the problem were concretely shown before they actually started on the problem.

Presentation of Data

Table 3.7.1. Means and standard deviations gradewise as well as sexwise for the various sub-samples

S. No.	Grade	Sex	Mean	Standard deviation
1	2	3	4	5
1.	VI	Boys	.65	.73
		Girls	.35	.48
		Boys & Girls	.50	1.26

1	2	3	4	5
2.	VII	Boys	.65	.48
		Girls	.35	.48
		Boys & Girls	.50	1.00
3.	VIII	Boys	.53	.48
		Girls	.55	.50
		Boys & Girls	.45	.93
4.	IX	Boys	1.30	1.19
		Girls	.55	.50
		Boys & Girls	.93	1.99
5.	X	Boys	1.5	.87
		Girls	1.7	1.10
		Boys & Girls	1.60	1.99

Summary of Results

Let us now summarise the main findings on this problem. These are:

1. Only one fifth pupils of grade VI experienced difficulty in reading the problem. Their I.Q.'s have ranged over 30 points. Most of their difficulties lay in distinguishing between cm and cc (cubic cm), writing dimensions of the cube but in wrong answer spaces, using available figures within the body of the question with little arithmetical operations made on them, and if not centimetre, then perhaps a kilometre will do.

2. Taking an overall view, the performance on this problem has been quite unsatisfactory for 40.5 per cent of the pupils were not in a position to tackle this problem. Whereas grade means increase with age, still the average performance on this problem in the case of grade X pupils is slightly above half the maximum score on it. Except a small fluctuation in grade X, mean performance favours boys rather than girls throughout the grades.

3. Throughout all the grades, the problem is failed over a wide I.Q. range within each grade and also across the various grades. This finding also applies to part of the problem solved successfully.

4. Except negligible fluctuations, all pupils of grades VI, VIII as well as girls of grade IX could not determine successfully the level of water reached on the immersion of the three rectangular cubes. They failed to utilise the given hint. Further, the evidence indicates that it is very difficult to apply a small piece of information, the decline in percentage being as large as 49 per cent on a simple test item like determining the volume of three cubes when the volume of one

of the cubes has already been successfully calculated. A slight change in context almost reduces the number of the successful pupils to one sixth of those who were successful on the past step.

5. Failure to understand the various elements of the problem causes all sorts of errors. Various figures are reproduced unreflectively and if judged incorrect, may be corrected. Generally speaking, they look for what sort of arithmetical operations can be carried out to reach the final answer. It is possible to obtain correct answers under certain conditions by using wrong thought processes. Moreover, it is very disturbing to point out that even a very simple test item has attracted a large number of erratic responses. For example, the number of errors committed on processes 46, 47 and 48 are 24, 6 and 19 respectively. It is only the dominant error on process No. 48 which undergoes a multiple hump. It may be pointed out that this is not an isolated case. Other errors committed by less than 15 per cent of the pupils have appeared but their frequencies are too small to be taken notice of for they appear, disappear, reappear or persist across grades, showing mini-humps.

6. Lastly, let us see how this problem correlates with other outside variables. It correlates positively and significantly with all the variables except the following: understanding the problem, confidence in the problem, interest in the problem, positive summation series problem and generalisation to algebraic symbols (summation).

8: COUNTING MAXIMALLY RECTANGLES PROBLEM

The Problem

Have a look at the diagram given below. Count the maximum number of rectangles that you can make.

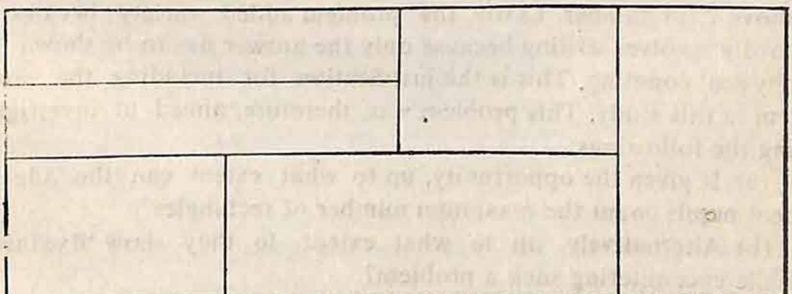


Fig. 3. Posing the problem.

Manner of Presentation and Scoring

The problem was presented as mentioned above. No hint was given except the frequent reminder that each of them should have a look at the diagram as differently as he could, counting in the process, as many rectangles as possible. Here, square could also be regarded as a rectangle. Tilting the figure in any direction was allowed. It was also stressed that no attempt should be made to draw extra lines.

It is easy to see six rectangles. On second reflection, it becomes, seven, considering the whole rectangle. On third and fourth reflections, the number of rectangles goes up to eleven.

Giving 6 or 7 rectangles	0 score
Giving 8 rectangles	2 marks
Giving 9 rectangles	2 more marks
Giving 10 rectangles	2 more marks
Giving 11 rectangles	2 more marks
	Total score = 8 marks

Elements and Aims of the Problem

While collecting problems from B.Sc., science students, two interesting problems appeared. One was on counting the number of triangles and the other on counting the number of rectangles maximally, using the same approach. The latter was preferred because it turned out to be simpler during the try out. This problem demands keen observation, seeing the various arrangements of the rectangles by overcoming 'fixedness' and in short, using insight or repeated restructings, leading finally to the counting of rectangles above 7, in number. Lastly, the problem added variety because it hardly involves writing because only the answer has to be shown by physical counting. This is the justification for including the problem in this study. This problem was, therefore, aimed at investigating the followings:

(a) If given the opportunity, up to what extent can the adolescent pupils count the maximum number of rectangles?

(b) Alternatively, up to what extent do they show 'fixedness' while encountering such a problem?

(c) What errors do they make while solving this problem?

Presentation of Data

Table 3.8.1. Means and standard deviations gradewise as well as sexwise for the various sub-samples

S. No.	Grade	Sex	Mean	Standard deviation
1.	VI	Boys	.40	1.02
		Girls	.70	1.92
		Boys & Girls	.55	3.19
2.	VII	Boys	0.00	0.00
		Girls	2.5	3.28
		Boys & Girls	1.25	5.27
3.	VIII	Boys	.6	1.80
		Girls	1.8	2.23
		Boys & Girls	1.2	4.78
4.	IX	Boys	2.3	3.42
		Girls	1.3	2.63
		Boys & Girls	1.8	6.61
5.	X	Boys	3.4	3.85
		Girls	2.8	3.70
		Boys & Girls	3.1	7.59

Summary of Results

The main results on this problem indicate:

1. It is very disappointing to note that a large number of pupils (72.5 per cent) are not in a position to restructure the field despite repeated reminders. To put in other words, they see the rectangle as a fixed entity with the consequence thas they fail to see that any one of the lines of any one of the located rectangle is also a part of another smaller or larger rectangle. Failure to restructure the diagram is colossal. The gradewise percentages are still higher as shown below:

	VI	VII	VIII	IX	X
Boys	85 %	100 %	85 %	60 %	50 %
Girls	85 %	60 %	75 %	75 %	50 %
Boys & Girls	85 %	80 %	80 %	67.5 %	50 %

These percentages show decline with age which is an expected finding. If this is treated as error, boys show a hump in their 'fixedness' in grade VII which persists in grade VIII as well. The same phenomenon persists in girls a year later and then declines. In grade

X, boys and girls are equally affected, there being no sex differences.

2. It is interesting to note that average performance on this problem favours girls rather than boys in grades VI to VIII. In the last two grades, the position just reverses. Except a single fluctuation in grade VIII, grade means increase with age. There is a distinct dip in performance for boys in grade VII and for girls in grade VIII. The performance on this problem is poor because grade X pupils have not been in a position to achieve even a half of the maximum score of this problem. Further, in regard to seeing the problem a bit differently, the percentage of pupils showing partial as well as full mastery on this problem is quite low as shown by the percentages given below:

	VI	VII	VIII	IX	X
Partial solution	12.5%	10%	12.5%	17.5%	20%
Full solution	2.5%	10%	7.5%	15%	30%

It is an expected finding that both percentages for partial as well as full solutions show an increasing trend with age. It is strange to point out that three quarters grade X science students find it difficult to see and re-see the problem even while frequently remained of exploring that opportunity. It is perhaps a typical phenomenon during adolescence as well in which students regard rectangle as a fixed entity: and once seen that way, they utterly fail to see one of its lines becoming a part of another smaller or larger rectangle. In the phraseology of Gestalt psychology, the given structure is as good as the prevailing conditions allow it, which may depend upon the particular conditions of viewing or reviewing at the given moment in which generous time limits as given in this study may not help. It is perhaps of this phenomenon which Max Wertheimer hinted at while investigating thinking of young students through problems both traditional and invented while commenting upon deceptive achievement. To quote Max Wertheimer:

> In addition to the specific structural experiences which we have in facing a problem—experiences which refer to structural perception, to observing the results of trials, etc.—there are many general features in our world which generally play an enormous role in our dealing with objects and which do so specifically in the concrete steps required for the solution of this concrete geometrical problem. They are so obvious that most of us do not think of them explicitly. Indeed, the reader may be shocked

even to see mentioned, for example, that in shifting the triangle from the left to the right, no change occurs in the size or the form of the triangle; that in doing so, no change occurs elsewhere in the figure, no contraction or expansion in other parts; that objects like parallelogram, etc., have their constancy, are not changed in size by drawing lines in them; that stated equality of some separate lines or angles secures equality of whole figures in distant places; that cutting into parts and rearranging them in an actual operation does not change the area; even that mere thinking operations—stating equalities, etc. —in no sense change the materials—, and so on [5].

3. Throughout the grades, the problem is failed over a wide I.Q. range both within each grade and also across the various grades. A few frequencies in the various cells indicated that problem is solved to a limited extent over a wide I.Q. range both within and across the various grades.

4. Sample responses, when analysed, reflected the growth of solution as follows: making comments on the problem; guesing the answer with hardly any support from the problem; guessing checked against the demand of the problem; drawing additional lines with a view to obtain more rectangles as well as an extra square or rectangle and making small squares in one of them; making measurements; and lastly, finding area of the rectangle. It appears that when confronted with a problem coupled with a feel that answer to the question needs a change, they change answers to suit more the requirements of the experimenter than the requirements of the problem. This finding is similar to the one obtained by John Holt in How children Learn where the correct answer given by the pupil depends more on the position of the teacher in the classroom rather than on his understanding [6].

5. Lastly, let us see how this problem is correlated with other variables. It correlates significantly with all the variables except the following: intelligence, school adjustment, understanding the problem, felt difficulty of the problem, confidence in the problem and interest in the problem. Interesting enough, it correlates insignificantly with the following two schemes of thought only, namely, using two digits at a time and using four digits at a time. The reason for this may be that possibly aspect character (or 'fixedness') is involved in this problem.

9: (COMBINATORIAL) DIGITAL PROBLEM

The Problem

You are given four numbers, namely, 6, 7, 8 and 9. Make as many digits or figures out of them as you can.

(i) 67 (ii) (iii) (iv) and so on.

Presentation of the Problem

The problem was presented as mentioned above. One of the solutions was given, that is, 67. Every pupil was asked to read and re-read the problem until he understood it. It was only stressed that each one of them could make as many combinations as he could think of—without multiplying and dividing the various digits. Only a few pupils from grade VIII asked whether they could multiply or divide. They were asked not to do this. No other hints were given.

Scoring

This problem utilises three distinct processes of thought when analysed logically. These are: Using two digits at a time (12 possible combinations out of which only one is already supplied); Using three digits at a time (24 possible combinations); and Using four digits at a time (24 possible combinations). Each suggested combination carried a mark each. So the maximum score carried by this problem theoretically speaking is $12+24+24=60$ marks. a very large figure. So a weighted score was used:

S. No.	Number of combinations	Weighted score
1.	1— 4	1
2.	5— 8	2
3.	9—12	3
4.	13—16	4
5.	17—20	5
6.	21—24	6
7.	25—28	7
8,	29—32	8
9.	33—36	9
10.	37—40	10
11.	41—44	11
12.	45—48	12
13.	49—52	13
14.	53—56	14
15.	57—60	15

Max. score = 15 marks

Elements and Aims of the Problem

On its very surface, this problem is very simple because it deals with four small individual numbers, lying between 5 and 10. It involves on the part of pupils seeing the individual number in isolation and in relation to the other numbers in all possible combinations, that is, in sets of two, three and four numbers giving in all sixty different combinations. It is at its best a simple modification of Piaget's experimental problem on combinations of coloured and colourless chemical bodies in which the beakers have been replaced by numbers, a more familiar situation to children [7]. Piaget and Inhelder say:

> The formation of propositional logic which itself marks the appearance of formal thought depends upon the establishment of combinatorial system. . . . The structured whole depends on this combinatorial which is manifested in the subjects potential ability to link a set of base associations or correspondence with each other in all possible ways to draw from them the relationship of implication, disjunction, exclusion [7].

In this context, this problem is aimed at investigating the following:

(a) Up to what extent can they structure this problem?

(b) Up to what extent can they exhaust all the possible combinations within a given and across allied categories which are supposed to exist logically?

(c) What are the most difficult combinations judged by the pupils?

(d) What errors do they make while tackling this problem?

(e) Taking on overall view, up to what extent are the various categories filled in?

Presentation of Data

Table 3.9.1. Means and standard deviations gradewise as well as sexwise for the various sub-samples

S. No.	Grade	Sex	Mean	Standard deviation
1	2	3	4	5
1.	VI	Boys	2.65	2.22
		Girls	2.70	2.33
		Boys & Girls	2.67	4.55
2.	VII	Boys	2.85	2.61
		Girls	2.05	1.24
		Boys & Girls	2.45	4.17

1	2	3	4	5
3.	VIII	Boys	5.70	3.99
		Girls	5.00	1.84
		Boys & Girls	5.35	5.51
4	IX	Boys	5.70	2.85
		Girls	5.10	2.95
		Boys & Girls	5.40	5.36
5.	X	Boys	7.25	2.26
		Girls	8.30	2.14
		Boys & Girls	7.78	4.57

Summary of Results

This problem had too many responses which presented its own characteristic problems of tabulation. The main results on this problem indicate:

1. Mean performance increases with grade. It favours boys rather than girls except in grade X. In fact, girls try hard to equalise their performance with boys and surpass them in grade X.

2. Single aspect character or polarisation in thinking is noticed which affects both boys and girls. It appears to be a temporary affair for it completely disappears in the closing grades. Hump effect is suspected.

3. Contrary to Piaget, adolescent pupils in this group are not in a position to exhaust all the possibilities. Except in few cases in the closing grades, systematic attack on the problem is hardly perceptible.

4. Two-digit category is filled in first and the three digit and four digit categories are filled in later on, the extent of their getting filled in being: 62.59 per cent, 23.12 per cent and 21.93 per cent. The illustrated step appears to have assisted pupils in filling in the first category.

5. As judged by the various frequencies which are quite large, it is easy to conclude that three categorised processes of thought, each taken separately, are failed or solved successfully over a wide I.Q. range not only within the grades but also across the various grades, as well.

6. The following combinations were found quite difficult:

(i) 86, 97 and 96 on two digit combinations given by 55 per cent, 55 per cent and 57.5 per cent of the pupils.

(ii) 896, 698, 769, 968, 796, 869, 986 and 697 on three digit

combinations were given by 11 per cent, 12 per cent, 12.5 per cent, 13 per cent, 14.5 per cent, 17 per cent, 19 per cent and 19.5 per cent of the pupils.

(iii) 9687, 8697, 6897, 7968, 7986, 9786, 8679, 9768, 8796, 8769, 8976, 8967 and 7698 on four digit combinations were given by 13 per cent, 13 per cent, 14 per cent, 14.5 per cent, 14.5 per cent, 16 per cent, 16.5 per cent, 17 per cent, 18.5 per cent, 19.5 per cent, 20 per cent, 22 per cent, and 20.5 per cent of the pupils.

7. Except a few fluctuations, especially due to polarisation in thinking, each combination is mastered increasingly as the pupils move into higher grades.

8. Contrary to Piaget, adolescent pupils commit a large number of arbitrary errors when there is failure to accept the main demands of the problem. Whereas there is general decline with increasing grade, boys are attracted more by the extraneous considerations than the girls. If actual combinations are counted physically which have no business to be there for they are based upon digits not given in the problem, it appears that these errors appear over a wide I.Q. range not only within a given grade but also across grades. These errors are, except occasional fluctuations here and there, not blind but manifest distinct modes of reasoning. Seven distinct types of reasonings were noticed which perhaps on maturity try hard to discover the basic structure underlying the problem.

9. Several interesting types of errors appear. First, if the supplied combination is again given which it is not necessary to give; and if it is regarded as error, it undergoes a hump. Secondly, resting points on combination other than the supplied one ought to have decreased with grade, they, on the other hand, also undergo a hump. Thirdly, if number of errors committed on three categories are counted gradewise and within the grade sexwise, these errors also undergo a hump. In the case of boys, errors undergo a hump in grade IX and in case of girls in grade VII and X (a bi-hump). When errors are pooled, they again undergo a hump in grades VII, IX and X (a tri-hump). Fourthly, the number of pupils failing to give a single combination on two digits, three digits and four digits combinations again undergoes a hump.

10. It is of interest to mention in passing about the relationship of this problem to other variables. At the usual level of significance, it correlates significantly and positively with all the problems as well as the schemes of thought. It correlates insignificantly

with the following variables: social adjustment, understanding the problem, felt difficulty of the problem and interest in solving the problems. Interestingly enough, it correlates more with grade than with intelligence, the corresponding correlations being .6144 and .2464 respectively.

10: QUESTIONS INVITING WRONG ANSWER PROBLEM

The Problem

Read the questions given below and then answer them carefully.

S. No.	Question	Process No.	Scoring key			Maximum score
1.	A blind man with one eye can see up to a distance of 100 ft. How far can he see with two eye''	54	0	200	ft.	2
			1	110	ft.	
			2	0	ft.	
2.	There are eighty birds sitting on a tree. A hunter came thereby and shot dead two of them. How many birds are now left on the tree?	55	0	78		1
			1	0		
3.	It takes two minutes for a handkerchief to dry up in the sun. How much time will it take for the twenty to dry up when placed in the sun at the same time?	56	0	40	minutes or any other time	1
			1	2	minutes	
4.	It takes 10 minutes for a boy to reach his school. How much time will it take for ten boys if they start for the school at the same time?	57	0	100	minutes or any other time	1
			1	10	minutes	
5.	A man crosses a stream in a boat in ten minutes. How much time will it take for five men to cross the same stream in the same boat?	58	0	50	minutes or any other time	1
			1	10	minutes	
6.	Ram has four friends. Their names are Shyam, Mohan and Jahaz. Find the name of the fourth friend.	59	0		Ram, Shyam & Jahaz.	2
			2		Suspending judgement	

S.No.	Questions	Process No.	Scoring key		Maximum score
7.	A stick is 10 cm long. It is cut by a cm per minute. In how much time will it be cut into 1 cm pieces?	60	0 10	minutes or any other time	1
			1 9	minutes	
8.	A boy went inside a dark room. There was a heap of red & yellow socks inside it. How many minimum socks should he pick up with a view to get a pair of the same colour?	61	0	Any other number	2
			2 3	socks	
9.	Suppose you are a driver of a bus. You have fifty passengers in it. At one place 15 passengers board the bus and 15 get down from the bus. At the other place again 5 board the bus and 5 get down from it. Please tell:	62			
	(a) How many passengers in all are there in the bus?			Not scored because everyone did it.	
	(b) What is the name of the driver?		0	Any other name	
			1	Own name	1
10.	Suppose a donkey has two horns. How many horns in all have eight donkeys?	63	0	No horns	
			1	If—then statement	
11.	Suppose there are some ducks swimming in a single file under a bridge. Two are in front, two in the middle and two at the rear. How many ducks are there in all?	64	0	Any other combination	4
			4	Four ducks	
12.	A cow is standing beside a tree. A cord of 50 cm is tied around her neck. Tell how far can she go for eating the grass.		0	Any other answer	1
			1	can go any where	
				Maximum score = : 17	

Manner of Presentation

These questions were presented as mentioned above. No other hints were given. All pupils were, however, allowed to read and re-read the various questions. They were also allowed to change their answers which were scored using the scoring key mentioned above. Responses strictly influenced by the content of the question carried 0 mark.

Elements and Aims of the Problem

As suggested by Koffka and also out of sheet interest, this problem containing 12 funny items was included in this study [9]. These intentionally invited wrong answers especially from those whose general grasp over the situation was poor. For this reason this category was called: failure to grasp the essence of the problem in which higher score showed the better grasp on the problem (later named as one of the schemes of thought). Our experience with these 12 items of the main problem showed that pupils became extra careful while tackling them. They, in fact corrected their previous responses as they went along solving them. This problem also lessened their boredom, if any, because it evoked, as they put it, unusual sort of thinking little subject to usual four fundamental rules of arithmetic for their mechanical application invariably gave wrong answers.

Our objective here was quite different. According to Piaget, children's thinking at their concrete stage of mental development is governed by reality of the situation or is tied to the concrete situation which, however, is not the case with pupils at the higher stage of mental development, that is, at the formal stage. Why? Because they are able to escape from the concrete situation: their thinking is governed by possibilities rather than realities of the physical situations. This qualitative changing in thinking helps them to see the same problem from several vantage points for their thinking has become propositional. To put in other words, they are influenced more by the form of the problem rather than its content. While doing so, they also master the tendency: to criticise or make comments on the problem which, however, was the case at the earlier stage [7, 10]. It was to explore this hypothesis that this problem was included in this study.

Presentation of Data

Table 3.10.1. Means and standard deviations gradewise as well as sexwise for the various sub-samples

S.No.	Grade	Sex	Mean	Standard deviation
1.	VI	Boys	3.9	2.32
		Girls	3.4	2.96
		Boys & Girls	3.65	4.81
2.	VII	Boys	4.15	2.15
		Girls	3.85	2.89
		Boys & Girls	4.0	5.99
3.	VIII	Boys	6.85	2.02
		Girls	4.75	4.00
		Boys & Girls	5.8	4.97
4	IX	Boys	9.85	2.44
		Girls	6.25	3.49
		Boys & Girls	8.05	7.02
5.	X	Boys	11.55	12.41
		Girls	8.00	3.87
		Boys & Girls	9.77	18.73

Table 3.10.2.—Errors committed, number of errors committed and gradewise distribution of incidence of content influence in percentages on processes 54 to 65 of problem No. 10

S. No.	Process No.	Errors committed	No. of errors	Incidence of content influence in per cent				
				VI	VII	VIII	IX	X
1.	54	200, 10, 20	3	40	35	22.5	17.5	—
2.	55	78, 68-78, 88-78 About 10-12, 40, 80, and about 1-2	7	70	57.5	42.5	15	7.5
3.	56	34, 40, 4, 21, 18 100, 4 hankies, 2 hankies, 40 ft 20	10	45	52.5	22.5	42.5	2.5
4.	57	20. 100, 1 hour, 40 seconds, 48, 100 boys	5	42.5	45	22.5	22.5	2.5
5.	58	50 minutes, 25 minutes, 15 minutes, and one minute	4	42.5	50	15	12.5	2.5

S. No.	Process No.	Errors committed	No. of errors	Incidence of content influence in per cent VI	VII	VIII	IX	X
6.	59	Ram, Shyam, Mohan and Jahaz, Any other name	5	100	95	90	90	82.5
7.	60	Minutes 10, 5, 1, 1 cm, 100 cm, 10 cm, 21, 20, Restating the question. It will decrease at the rate of 1 cm sec.	9	75	97.5	80	70	32.5
8.	61	2, other numbers like: 47, 40, 46, 20 10, 5, 4, 1, 0, Pick two colours or many, one hour, Switch on the light, Can't say, use mixed one, can't see in the dark room.	9	97	95	87.5	75	75
9.	62	Suppose any name Can't say. You. No names. Suppose, you are	5	80	62.5	50	20	12.5
10.	63	0, 2, 8, 1	4	40	30	47.5	55	45
11.	64	6,82, 3, 7, 9, 24, 0, 5, Can't say, many, some	12	92.5	100	95	72.5	50
12.	65	50 cm, 55 50×4, 400, 5-50, 2 11π, $11\pi^2$ 100, 7 metres, 10 cm, 1 metre, 5 cm, 3 cm, 48 cm, 52 cm, 2500 square	17	97.5	100	97.5	82.5	82.5
		Overall incidence		68.4	68.22	56.04	45.42	32.92

Table 3.10.3. Dominant error and its gradewise distribution in percentages on processes 54 to 65 of problem No. 10

S.No.	Process No.	Dominant error	VI	VII	VIII	IX	X
1.	54	200	40	35	17.5	17.5	—
2.	55*	78	70	47.5	37.5	15	7.5
3.	56	40	40	42.5	22.5	12.5	2.5
4.	57	100	42.5	42.5	20	12.5	2.5
5.	58	50	62.5	45	15	15	2.5
6.	59	*Given name*	85	82	67.5	70	82.5
		any other name	15	12.5	25	20	5
7.	60	10 minutes	67.5	70	45	45	32.5
8.	61	2	77.5	25	20	25	12.5
9.	62*	Any other name	80	62.5	50	20	12.5
10.	63	0	27.5	20	42.5	52.5	37.5
11.	64	6	35	87,5	67.5	57.5	32.5
12.	65	50	90	82.5	82.5	77.5	72.5

*No hump effect

Summary of Results

1. The gradewise means increase with age which show that pupil's grasp on the problem, as a whole, increases as they move into higher grades. The sexwise means favour boys rather than girls throughout the grades.

2. Contrary to Piaget and as judged by high incidence of content influence, adolescent pupils are affected by the content of the question over a wide I.Q. range not only within grades but also across grades. Taking an overall average of the influence of content, the extent of influence declines with grade, a finding consistent with Piaget, the respective gradewise percentages being: 68.54, 68.33. 56.04, 45.42 and 32.92 from grades VI to X (see Table 3.10.2). Even in the closing grade, the percentage is on the high side which suggests a fruitful hypothesis: It is possible to evoke a wide range of concrete behaviour at the formal stage by choosing a suitable problem. To put in other words, when the choice of the problem is appropriate, the adolescent pupils do not hesitate at all to bring into the problematic situation several extraneous considerations.

They then cannot look at the problem at a distance or cannot separate themselves from the situation. To pinpoint, when the supposedly availabie scheme of thought fails to solve the problem, the inferior one comes into play, bringing in its train, comments, criticisms and other arbitrary errors into the problematic situation only with the restriction that they show gradual mastery in grasping the essence of the problem with age. It is, therefore, least surprising that any item of the main problem is solved successfully or failed over a wide I.Q. range not only within the individual grades but also across the grades, as well.

3. The adolescent pupils think hard while solving even questions needing restricted thought. This is confirmed by the fact that several errors unexpectedly appear when they fail to accept the problem, their number ranging from 3 to 17 on item No. 1 and 12 respectively (see Table 3.10.2). To concretise, several ideas compete hard when their thinking is influenced wholly by the content of the problem. But there is one constraint here, that is, when they suspect something fishy, they become extra careful in suggesting or reconsidering a well considered answer which could be quite mathematical (and even still go wrong). Examples are:

(i) If a blind man tries hard, he can see easily up to 10 to 20 ft.

(ii) Either there are 78 birds left or say between 68-78 because birds after all do not know counting. Secondly, their number well could be between 78-88 because after the shot is over, a few more birds may come to the tree for rest. They thus join their friends. Thirdly, all of them may not fly because still 1-4 birds might not even try to fly.

(iii) Time required for drying up is given not only in minutes (mechanically calculated) but also in number of hankies, centimetres (distance) and hours.

(iv) The fourth friend of Ram can have any name because he has to have (possess) a name (The adolescent pupils ought to be in a pos tion to suspend judgement when there is no clue to find the name in the body of the question itself.)

(v) When it comes to the picking up of minimum number of socks, several answers appear, interestingly enough, one of them being zero. Large number up to 46-47 or many socks are also given.

(vi) (Suppose you are the driver) They find it difficult to see themselves in the position of a driver. Instead, they supply any name of any driver known to them or suggest one of their own free will.

(vii) (How many horns have eight donkeys)? It has puzzled many. It has attracted four different responses. First, eight donkeys have no horns because a donkey has no horns, the argument being: Why should I suppose wrong things when I can physically see? Secondly, the answer is 2 because only one donkey has two horns. "How can I suppose for the remaining seven donkeys"? Thirdly, the answer is 8 because the number of donkeys in the question is 8, a jolly good example of equality of opportunity. Fourthly, the answer is '1' because a donkey has a horn. It does not matter whether it is one or two.

(viii) How many ducks are swimming under the bridge? If they see eight, they cannot see six. If they see six, they cannot see four (the correct answer). There are some ducks and as many ducks which are to be found out, an illustration of rephrasing the question. The ducks are swimming under the bridge and no one is in fact, coming out and hence the answer is '0'.

(ix) (How far can the cow go?) Lastly, several interesting answers have appeared on this question. Let us illustrate. First slightly more than 55 because the cow has its own length, or she can strain a bit. Secondly, it is 50×4, 4 being the side of the square or rectangular field. Thirdly, it is $2 \pi r$ or πr^2, r being the radius if the cow turns all around. Fourthly, distance is given in cms, metres and square cms, as well.

4. If the above responses are regarded as errors because they arose simply from the direct influence of the content contained within the body of the question without appreciating the main demand of the 12 individual questions, each taken in isolation, it is found that errors on nine of them undergo a hump, three exceptions being the errors on processes at S. Nos. 54, 55 and 62 (see Table 3.10.3). It is, therefore, hypothesised that, cumulatively speaking, errors arising due to strict consideration of the content within the body of the question without appreciating the form ought to undergo a hump.

In continuation, all the twelve processes of thinking brought forth 12 dominant errors out of which 10 suffered a hump each of varying nature, the two exceptions being the errors on processes at S. No. 55 and 62. On one of these ten processes, namely, process No. 59, two dominant errors appeared and each suffered a hump. These humps appear because they involve grasping the basic essence of the problem. The question: How many horns have eight donkeys? is a case in point. Donkeys have no horns but the answer to the question is governed by the word 'suppose' with complete unconcern for the absurdity of the answer! The resulting hump is shown as follows:

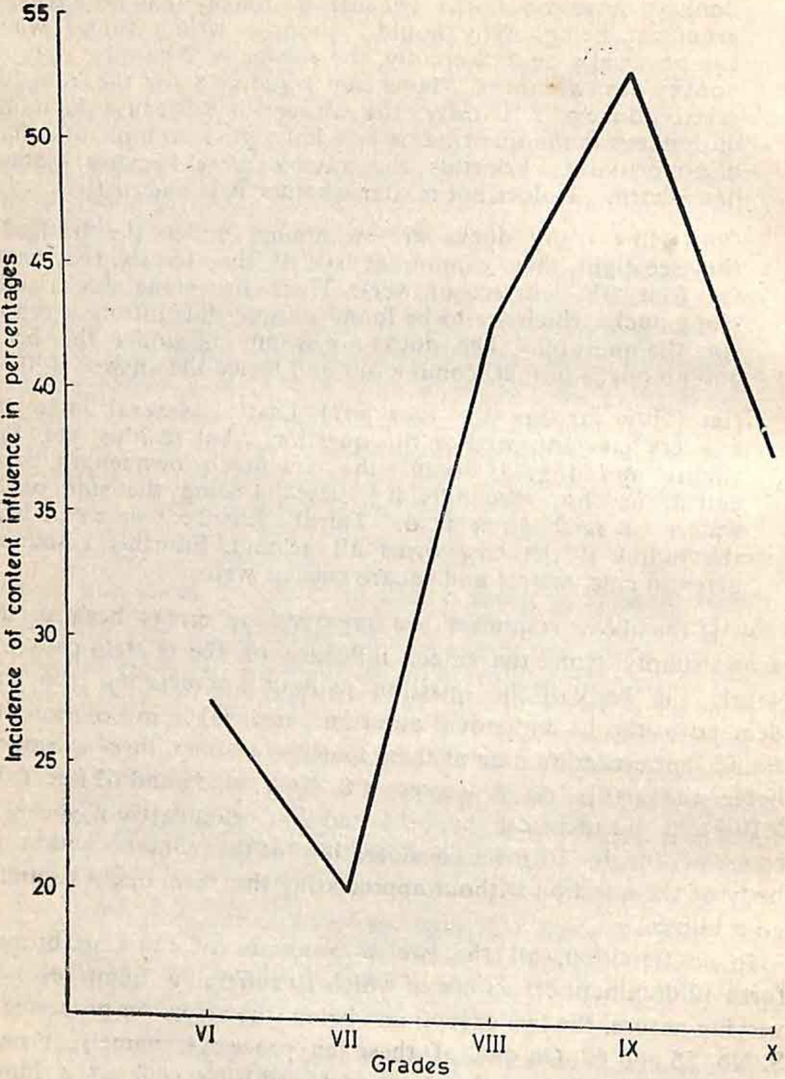


Fig. 4. Hump on process No. 63 (How many horns have eight donkeys?) of problem 10.

5. Majority of adolescent pupils cannot suspend their answer (judgement) when they tackle a defective or incomplete question, a question whose answer is not available for the needed information is missing within the question itself. This again shows that they are, largely speaking, influenced more by the content rather than

the form of the question (Question No 5, Process No. 59), the gradewise percentages of those who could handle a complete question are: 0. 5, 10, 10 and 17.5 respectively.

4. Failure to accept the problem results in all sorts of errors called arbitrary errors which, when pooled, suffer a hump of varying nature. Secondly, dominant errors as defined in this study also suffer hump of varying nature. This confirms another finding that more than 90 per cent of the adolescent pupils studying in grades VI to X are not in a position to suspend a judgement or judge the incomplete nature of the question.

5. Lastly, let us see how this problem correlates with other variables. As expected, it correlates positively and significantly with all the remaining problems as well as schemes of thought. It also correlates significantly with grade as well as intelligence. It correlates significantly but negatively with felt difficulty of the problem (due to reversible scoring). It is interesting to note that it correlates insignificantly with understanding of the problem, confidence in the problem and interest in the problem.

11: NINE DOT PROBLEM

The Problem

I have marked nine dots below. Have a careful look at them. Draw four lines in any manner you like in such a way that these lines touch all the dots at least once. In case you find that you have drawn more than four lines, then attempt again. You can try as many times as you like.

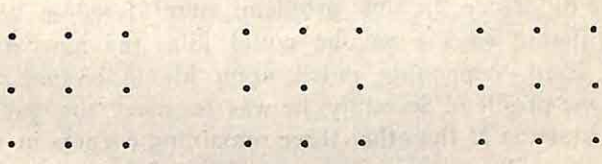


Fig. 5. Posing the nine dot problem.

Manner of Presentation

The problem was presented as mentioned above, allowing the lifting of the pencil. After every pupil have had sufficient number of trials on it, the problem was represented a bit differently. Starting from one of the corners, a specimen solution was given which was shown clearly but not explained. Then every pupil was asked to provide three other similar solutions starting at the other

three remaining corners. No other hints were given. The specimen solution as given using the top left corner is given below:

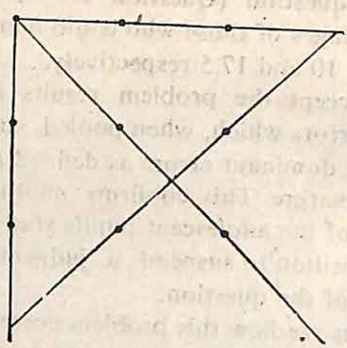


Fig. 6 A specimen solution utilising the top left corner.

Scoring

It was done in two ways. One mark each for three solutions obtained in any manner; and three marks each for suggesting a solution in accordance with the given example or illustration.

Maximum score = 12 marks

Elements and Aims of the Problem

This problem is said to have been used by the Gestalt psychologists for investigating thinking among superior university students in Germany. In the absence of any experimental data or evidence available on this problem, this problem in a slightly altered form, is in certain respects, similar to problem No. 8 for it also involves seeing and consequently structuring and restructuring the field with a view to meet the main demand of the problem. There is, however, one difference. In this problem, pupil freedom was allowed in two different ways. First, he could join the nine dots in any way he liked, depending much upon his individual manner of viewing the problem. Secondly, he was to meet the same requirement by starting at the other three remaining corners in accordance with the given illustration. Count was kept of the incorrect number of trials made by each pupil while tackling this problem. In short, this problem is aimed at investigating the following:

(a) If given the opportunity, up to what extent can adolescent pupils meet the demands of the problem?

(b) Up to what extent can they utilise the hint provided by the illustration without explanation in advancing the solution of this problem?

(c) What errors do they make while solving this problem.

Presentation of Data

Table 3.11.1. Means and standard deviations gradewise as well as sexwise for the various sub-samples

S. No.	Grade	Sex	Mean	Standard deviation
1.	VI	Boys	1.65	2.17
		Girls	0.00	0.00
		Boys & Girls	.83	4.49
2.	VII	Boys	2.4	1.80
		Girls	.3	0.87
		Boys & Girls	1.35	3.54
3.	VIII	Boys	1.85	1.91
		Girls	1.45	1.52
		Boys & Girls	1.65	3.47
4.	IX	Boys	3.05	2.38
		Girls	3.3	2.10
		Boys & Girls	3.68	4.55
5.	X	Boys	6.6	3.75
		Girls	7.5	.40
		Boys & Girls	7.05	7.78

Table 3.11.2. Acquisition of thought processes in terms of number of pupils gradewise as well as sexwise for the various sub-samples

Grade	Sex	Processes					
		66	67	68	69	70	71
VI	Boys	9	8	4	3	1	0
	Girls	0	0	0	0	0	0
VII	Boys	14	5	5	8	0	0
	Girls	2	2	2	0	0	0
VIII	Boys	12	10	9	2	0	0
	Girls	13	10	9	0	0	0
IX	Boys	20	20	20	4	2	1
	Girls	19	19	19	1	1	1
X	Boys	20	20	20	13	6	5
	Girls	20	20	20	12	11	6
Total		129	114	108	43	21	13
Percentages		64.5	57	54	21.5	10.5	6.5

Summary of Results

Below are summarised the main results:

1. As expected, the mean performance on this problem increases with grade. Except in grade X, the mean performance favours boys rather than girls throughout the grades.

2. Majority of pupils from grades VI to VII have failed to structure this problem, the gradewise percentages being 77.5 and 37.5. Seen sexwise, all the girls of grade VI and 90 per cent girls of grade VII could not structure this problem at all (see Table 4.11.2). Taking an overall view, it is safe to conclude that a given problem is failed over a wide I.Q. range within grades VI to VIII and solved partially over a wide I.Q. range in the remaining grades (see also Table 3.11.3).

3. When seen in terms of mastery over the various processes, it is found that 35.5 per cent of the pupils could not show mastery on the first thinking process of the motor variety, that is, S. No. 66. On processes 67 and 68, this percentage rose to 43 and 46, the overall rise being 10.5 per cent, on processes more or less of the same type but involving a different configuration. To put in other words, this meant that most of them, that is, 46 per cent did not possess insight into the solution of this problem. When illustration without explanation was shown, this percentage further rose to 78.5, 91.5 and 93.5 showing thereby that hint is of less assistance to a vast majority of pupils, the main victims being the pupils of grade VI to IX.

It is only in grade X that the illustration was utilised in suggesting the other three solutions to a varying extent by 62.5 per cent, 42.5 per cent and 27.5 per cent of the pupils across the three processes (69, 70 and 71) in which either the principle of solving remained the same or the form and content of the three consecutive test items remained more or less the same in relation to the provided illustration open to observation. Considering the pooled sample (N=200), the percentage drops from 54 on process No. 68 to 6.5 on process No. 71, the drop being as large as 47.5 per cent.

4. Pupils made several unsuccessful trials while tackling this problem. When added gradewise, the number of such trials were found to be: 568, 563, 370, 389 and 468. If those trials are regarded as attempts at grasping the essence of the structure underlying the problem and equated with errors as committed during problem solving, these errors like other errors of the earlier problems also suffer a partial hump as shown by the following curve:

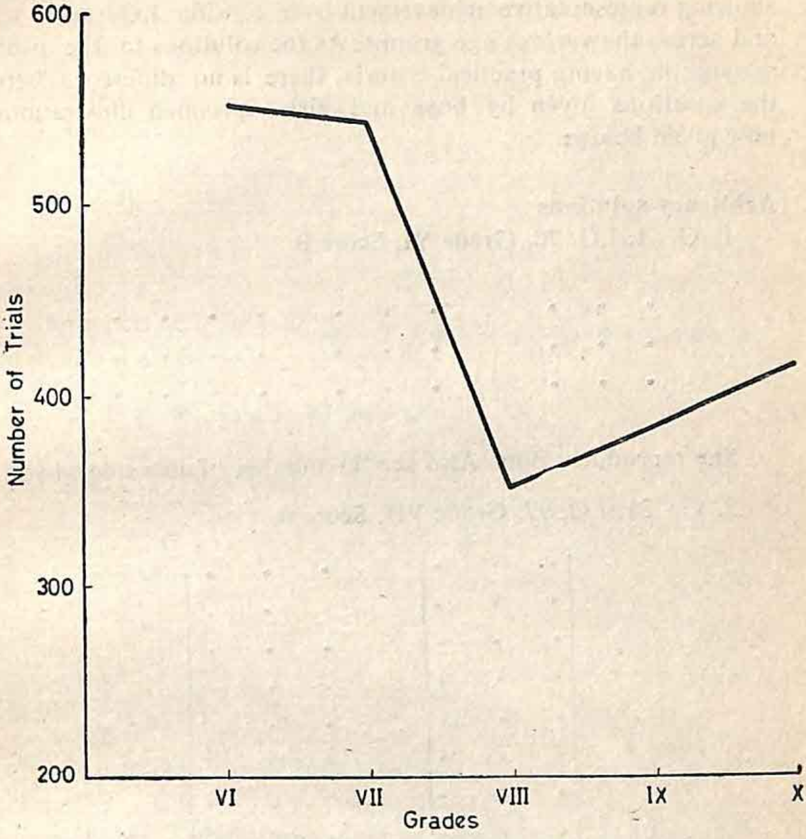


Fig. 7. Distribution of unsuccessful trials made gradewise.

5. This problem correlates more with grade than with intelligence, the corresponding correlations being $r=.6347$ and .1823. It correlates insignificantly with the following variables:

Home adjustment, social adjustment, understanding of the problem, felt difficulty of the problem, confidence in the problem and interest in the problem. With all problems as well as schemes of thought, it correlates positively as well as significantly.

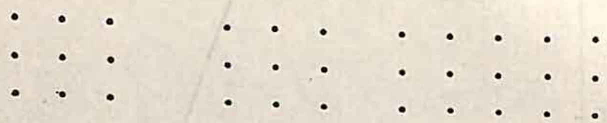
Sample Responses

Let us present below quite a few sample responses which attempt to trace the growth of the solution of this problem right from the arbitrary solutions (or blind structures) via transitory ones to the specific (or clear) solutions, that is, when the main demand of the problem is fully met. It may be mentioned that these arbitrary

errors or blind structures have come from most of the pupils showing representative achievement over a wide I.Q. both within and across the various age groups. As the solutions to the problem are specific having practical criteria, there is no difference between the solutions given by boys and girls. Specimen illustrations are now given below:

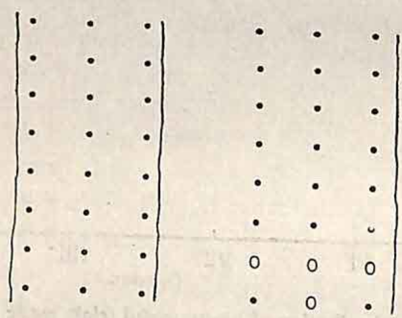
Arbitrary solutions

1. G 1, I.Q. 70, Grade VI, Score 0



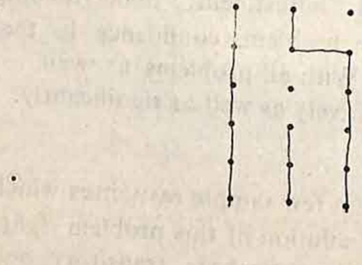
She reproduces dots. Also see the number of dots side ways.

2. G 31, I.Q. 97, Grade VII, Score 0



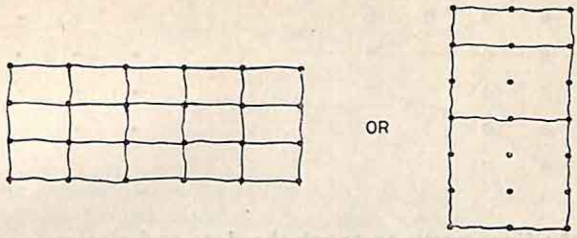
Encloses dots and circles.

3. B 6, I.Q. 82, Grade VI, Score 0



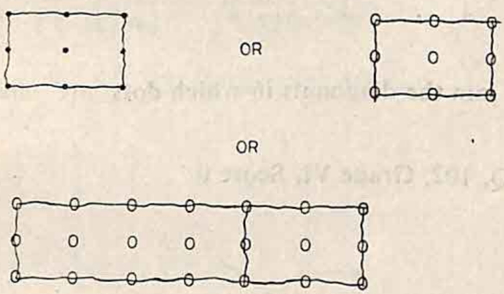
Touches more dots than required.

4. G 2, I.Q. 72, Grade VI, Score 0



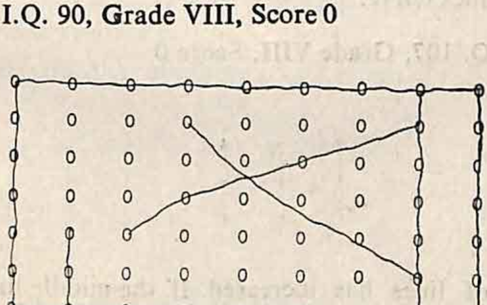
Forgets the number of dots. Dots are missed. Attempts the problem in two dimensions.

5. G 20, I.Q. 120, Grade VI, Score 0



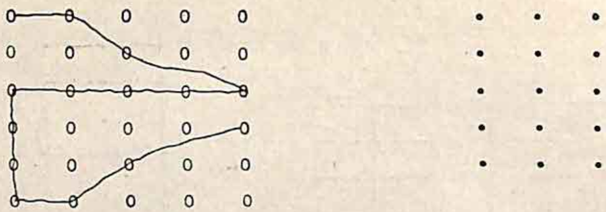
Central dot or circle is missed. Dots change into circles. More dots are used horizontally.

6. G 28, I.Q. 90, Grade VIII, Score 0



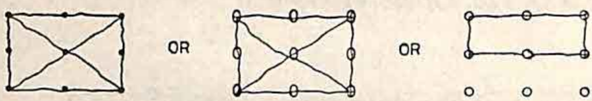
See how four lines are imagined: Structure into structure.

7. B 46, I.Q. 82, Grade VIII, Score 0



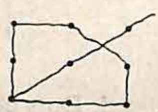
See how the lines curve to touch the dots and circles.

8. G 7, I.Q. 85, Grade VI, Score 0



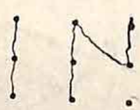
Tendency to join the diagonals in which dots are changed into circles.

9. B 13, I.Q. 102, Grade VI, Score 0



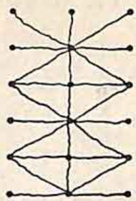
See how the lines curve.

10. B 35, I.Q. 107, Grade VIII, Score 0



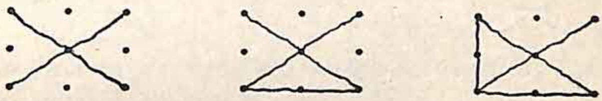
The number of lines has increased. If the middle line forming the letter 'N' is ignored, the problem is solved. But that is a break in the configuration.

11. B 53, I.Q. 102, Grade VIII, Score 0



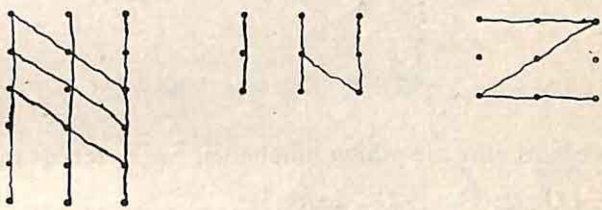
The solution is sought in varied configurations in connection with the hope a common line may appear.

12. B 1, I.Q. 70, Grade VI, Score 0



Two Lines Three lines Four lines but the dot is missed

13. B 47, I.Q. 85, Grade VIII, Score 0

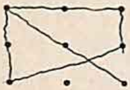


See how the solution is oriented to meet the demand of the problem by considering several dots at one go. The problem would have been solved, had he considered nine dots only using the same approach.

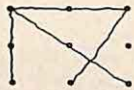
14. G 48, I.Q. 90, Grade VIII, Score 0

When this attempt fails, another interesting attempt is made in which another extra dot is imagined.

15. B 17, I.Q. 100, Grade VI, Score 0



OR



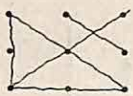
Extra dot is added and the given one is ignored. By slight curving one of the lines, the number of lines stays at four.

16. B 7, I.Q. 85, Grade VI, Score 0

OR

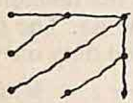
A good case of 'fixedness' when the position of nine dots is considered as if determining the boundary of the lines.

17. B 49, I.Q. 92, Grade VIII, Score 0

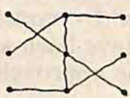


See another hard effort in which number of lines exceeds four.

18. B 63, I.Q. 75, Grade IX, Score 3

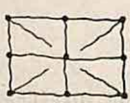


OR



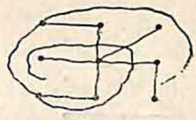
Trying hard to understand the structure.

19. G 61, I.Q. 70, Grade IX, Score 0



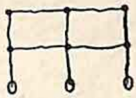
Suspends further effort.

20. B 80, I.Q. 120, Grade IX, Score 3



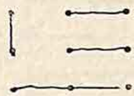
See how the lines curve to discover the structure.

21. B 62, I.Q. 72, Grade IX, Score 3



Failure to discover the structure results in dropping lines, like pins.

22. B 48, I.Q. 90, Grade VIII, Score 3

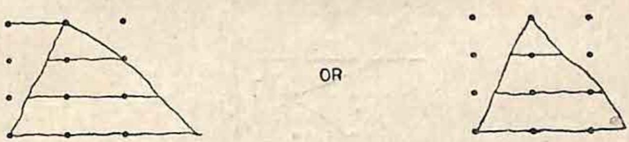


An interesting response based upon literal interpretation.

23. B 96, I.Q. 110, Grade X, Score 6

Dot is shifted to meet the solution.

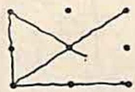
24. B 89, I.Q. 92, Grade X, Score 0



Dots are shifted to make triangles within a large triangle.

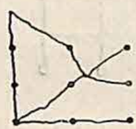
Transitory Solutions

25. B 9, I.Q. 92, Grade VII, Score 0



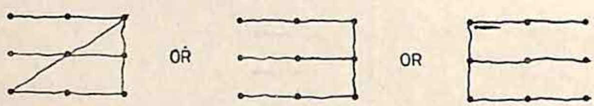
See how the lines curve as they go along.

26. B 11, I.Q. 97, Grade VI, Score 2



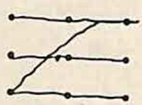
Dots must be joined.

27. B 10, I.Q. 95, Grade VI, Score 2



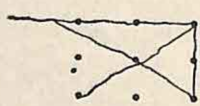
See the shift

28. G 72, I.Q. 100, Grade IX, Score 3



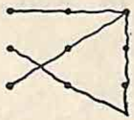
See how the central dots are missed.

29. G 76, I.Q. 110, Grade IX, Score 3



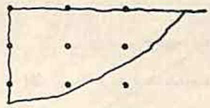
Only one of the lines is extrapolated.

30. B 69, I.Q. 92, Grade IX, Score 3



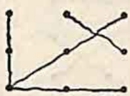
No effort but one of the lines is extended.

31. G 61, I.Q. 70, Grade IX, Score 3.



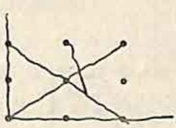
Two lines are extended but again the solution is missed.

32. B 22, I.Q. 72, Grade VII, Score 4



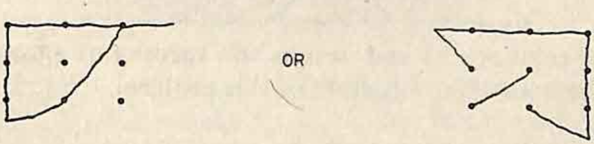
Nearer to the solution.

33. G 86, I.Q. 82, Grade X, Score 9



Lines are extended but structure is not discovered.

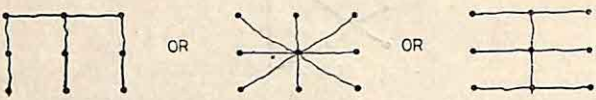
34. G 82, I.Q. 72, Grade X, Score 0



Hesitant to meet the demand of the problem. Also see how the problem is viewed.

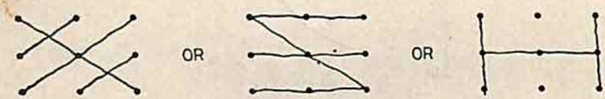
Specific Solutions

35. G 61, I.Q. 70, Grade IX, Score 3

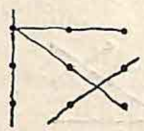


Demand of the problem is met.

36. G 68, I.Q. 90, Grade IX, Score 3

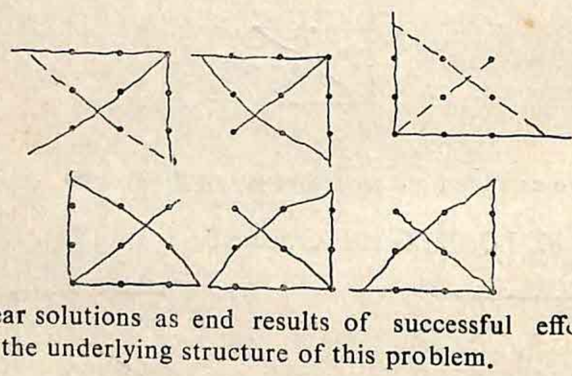


37. G 91, I.Q. 97, Grade X, Score 12



Tries to understand the structure and succeeds.

38. G 79, I.Q. 117, Grade IX, Score 12
39. G 95, I.Q. 107, Grade X, Score 12
40. G 97, I.Q. 112, Grade X, Score 12



See clear solutions as end results of successful efforts which reflect the underlying structure of this problem.

Discussion

Let us see now what these diagrams mean to us. What happens when pupils fail to structure this problem in accordance with its demands? Can we see or judge those attempts which lie midway

between the arbitrary attempts, that is, being blind to the structure in the phraseology of Max Wertheimer on the one hand and the exact or specific solution or clear solution on the other hand? The above mentioned specimen diagrams indicate the following stages constituting the solution of this problem with arbitrary attempts at the base of the problem. The characteristics of these attempts were:

(i) If you can't understand as well as solve even a bit of the problem, start reproducing dots. Children especially of grades VI to VIII do get interested in drawing dots for they have earlier played the Dots Joining Game at school when the teacher was away as well as at home on a holiday with their brothers, sisters and friends.

(ii) If dissuaded, they start making rectangles either horizontally or vertically. If dissuaded still further, they begin to number them. If considered necessary, depending upon the availability of time, they make few squares as well.

(iii) If further prompted to solve the problem, they really forget the problem, begin to work intensively on the same pattern, using the whole page.

(iv) Tendency then appears to draw the rectangle or square with middle dots remaining untouched (by the pencil). Square or rectangle is made again for the dots have to be touched once and consequently, diagonals go both with the squares and rectangles. Then the number of lines increases more than the permissible number (only four lines allowed).

(v) A shift in attack takes place. There is a tendency to draw two long lines vertically with middle dots missed or ignored. This does not help for the number of lines remains much less than the permissible four. Attempt is then made to make a square or rectangle with one side missing. The said form of the solution remains unacceptable because one of the dots at the side is missed.

(vi) If nothing works, one encounters another tendency to enclose the available dots within the two vertical lines and then the interesting information is given:

"My name is Anita."

(vii) (Pupil is then reminded to return to the problem). Another tendency appears: use quite a few of the large available dots for making a large sized rectangle. This is the outermost rectangle. Since several dots inside remain untouched, there is then the further tendency to make a large number of rectangles inside with one of the sides missing, if judged necessary.

(viii) Nature loves symmetry and so do children. If rectangle does not tempt, a square may. Action is then somewhat as follows:

Make a large square and use the inside dots for making four small squares. In order to restore sovereignty to the large square, provide diagonals to it. In this exercise, it hardly matters if the diagonals fail to meet at the centre or at the central dot.

(ix) Now another interesting behaviour appears. It is necessary to solve the problem because for them, some solution is better than no solution at all. If problem can't be solved with nine dots, why not use six dots? If this does not help why not add dots? If dot is not visible, put another dot adjacent to the one ignored. If dots do not help, why not change them into circles? In turn, if circles are found of no use or significance in the solution of the problem, make each dot a short line looking like 'l's. If in these attempts, it is found that the number of lines has increased beyond the permissible four, then attempt is made to drop either one of the diagonals or two sides of the rectangle or square. If squares and rectangles fail to solve the problem, attempt is then made to make triangles or parallelograms. If one large triangle does not solve the problem, then further effort is made to draw more sub-triangles within the larger one. When seen in a larger perspective or even configuration, it only amounts to drawing lines at the centre; and, a few other versions, where anyone of the inside dots becomes the main or minor centre of interest, depending upon the individual viewing of the problem.

(x) When varieties of diagrams like rectangles, squares, triangles or parallelograms (both complete and incomplete) fail to solve the problem, the demand of the problem is met in another interesting manner, that is, by curving or bending the lines with a view to join the nine points. This line of attack results in several zigzag figures for now every dot is seen not as static but a mobile one. Extra lines, not at all essential to the solution of the problem, may also be added as part of the carry-over effect of the past trials made. Lastly, only one option appears to be left, that is, interpret the problem too literally. This is done by drawing four isolated lines which join the nine dots for the demands of the problem are met with the consequence that the prospective reformulation of the problem becomes quite unnecessary.

What these efforts amount to? They amount to firing, taking a nuclear analogy, alpha particles, at the nucleus, which they cannot, for by their very nature at this stage of mental development: (their being blind to the structure). Their trials, therefore, remain at their

very best outside the peripheral solution. They must succeed for the problem is to be mastered. For achieving this, they either change the figure, then the ground or every possible variable judged of some assistance in that Gestalt situation. To illustrate, dots change to circles and circles, in turn, to 'l's, dots change quantitatively both more or less than nine in regard to position and orientation, quadrilateral figures taking their different forms with side(s) and diagonal(s) missing; and lastly, the very curving or bending of the lines touching or missing dots, etc. These efforts as the experience with this problem showed are not, really speaking, arbitrary or blind or clumsy but are intentional to discover the fundamental structure underlying the problem surrounding which a variable solution is available. This is seen clearly when all these figures change into transitional solutions in which awareness towards completness of the solution of the problem is implicitly present.

Examples are: extending the adjacent and then trying hard to place the dots on the arc; extending one of the lines and joining dots through a diagonal, and, if found necessary, drawing an auxiliary line to it (acting in the process, like Birbal who shortened the emperor's line by drawing longer line adjacent to it for the emperor's line according to the command must not be touched at all), encircling the dots by drawing lines not touching dots with a view to close the problematic situation structurally, using curved or inclined lines, seeing and seeking extrapolation but not making use of it when it comes to the solution of the problem; seeing the situation and attempting to grasp it but still hesitation to submit a solution possibly for fear or ridicule, and lastly, using lines in any manner (in zigzag ways). Why? To break the structure and build it again and again until the problem is solved, a behaviour which takes place over a wide I.Q. both within and across the various grades.

Educational Significance

The observations made above have sound educational significance. Max Wertheimer in his book on Productive Thinking has strongly criticised the traditional methods of teaching and learning employed by teachers and their students in turn, to show conventional and socially acceptable achievement in which, largely speaking, understanding, application, grouping, centring, reorganising, grasping the basic essentials, insight into the structure of the solution and variations in the general solution to meet the specific requirements of the altered problem are absent [11]. Through jokes,

concrete problems of playway variety and arithmetical as well as geometrical exercises, he explains his fundamental educational principle which when stated negatively implies that children need not remain 'blind to the structure' while tackling subject matter problems. Their thinking processes are not like individual match sticks staying in the match box for unlike them, they have their characteristic nature of assistance to each other while looking for sensible expectancies. Why? Because 'thought processes show a consistency of development' [11]. It is not necessary to teach the area of rectangle and parallelogram in isolation of each other for it is relational. Teaching should not destroy the 'old logical differences between essentials and inessentials'. In his view, the right answer need not be highly exact and specific if it is to be reconstructed by the child unaided. To quote Wertheimer while analysing the solution of the Bridge problem attained by deaf mute children:

> When negative performance occurs, whether in children or in grown ups, it need not be a sign of lower intelligence. Quite different factors may play a role: difficulties in the handling of bricks, awkwrdness, clumsiness. In some grown ups, there is the additional factor of dislike for being tested, of disdain for such problems, of being subjects in experiments, of being before an audience, etc. [11].

12: FORMULATING QUESTIONS PROBLEM

The Problem

Let us formulate questions.

Children of your age are very curious. Many questions of different types arise in their heads whose answers they do not know. For example, there was a student named Mohan of your age. He asked the following questions on the sun whose answers he did not know.

(a) Is the sun actually a ball of fire?
(b) Why does not the sun fall down on the earth?
(c) Would we have survived had there been no sun?
(d) What will be the temperature of the sun?

Questions like this may also be arising in your head. Now make as many questions as you can on:

(a) Cycle
(b) Cow

whose answers you do not know.

Manner of Presentation

Each pupil was asked to read and re-read the above question. He was allowed to seek any clarifications within the statement of the problem. There was no interruption once he started.

Scoring

One mark for each acceptable question. Here only those questions whose answers were too obvious were rejected straightaway from scoring. This gave score on fluency. While score for fluency was being obtained, it appeared that these questions could also be scored for flexibility as well meaning thereby the typology of questions starting with: What, Why, How, Is and other categories. Flexibility was scored for giving one mark each for all questions falling under one category. This gave us an additional scheme of thought.

Elements and Aims of the Problem

Not long ago, John Holt remarked: "We encourage children to act stupidly, not only by scaring and confusing them but by boring them, by filling up their days with dull, repetitive tasks that make little or no claim on their attention or demands on their intelligence" [12]. He further adds that most of the time they are engaged on dull tasks, result being that they hardly make use of their talents and tools because "before long they are deeply settled in a nut of unintelligent behaviour from which most of them could not escape even if they wanted" [12]. These remarks refer to the most expensive schools of the U.S.A. which he personally visited. These remarks not being casual, the true state of affairs is much worse in our country. It is so because the sole emphasis in our day-to-day teaching has been and is on underfeeding and overexamining; examination rather than evaluation and education; information in contrast to formation; facts in contrast to concepts; and stereotyped questions and answers in contrast to open questions and answers. At the same time, it is still believed that under certain conditions, children can be made to think and raise questions. They can as well be guided to answer them [13]. In two other studies, the writer also collected a large number of questions from the primary and middle school children studying in the Punjab, Himachal Pradesh and Rajasthan.

The results of these studies indicated that children ask all sorts of questions which stem practically from all the significant areas of human living. They naturally pick up much information incidentally from their immediate environment in which they live and acquire partial facts and concepts. In this process they encounter some

problems based mostly on direct observations and try to investigate them in their own characteristic ways at their own level of mental development. These two studies lacked: specifying a topic on which problems could be formulated; considering the typology of questions; and mental development. If evidence in this direction is established which turns out to be favourable, this informal approach to curriculum development may end in environmental studies. For example, a critical shortage in soap may present such a problem provided a wide awake teacher is on the look out of such problems in the immediate environment of his charges. To quote Johnson:

> Ninth grade students added soap solutions volumetrically (by means of medicine droppers) to samples of water. They compared results with great interest, noting corresponding results with certain samples and different results with other samples. Many problems close to their lives grew out of this observation: Were the samples of water the same? How could water be treated to save soap? Did service stations use distilled water for car batteries and others.
>
> Many separate projects resulted, involving water softening in the home, costs of soap and soap powders, and the like, and the reports made in class resulted in still other class activities [14].

Even backward children can be trained to think this way provided we are sensitive enough to capture the right type of centres of interest for them. Lastly, the main reason for including this problem in this study was to check the hypothesis whether ability to formulate problems quantitatively as well as qualitatively was related to other schemes of thought included in this study. In this context, this problem aims at investigating the following:

(a) What acceptable problems or questions are asked by the adolescent pupils?

(b) What is their performance in formulating problematic situations from grade to grade?

(c) Is there any relationship between the fluency and flexibility aspects of the problems asked?

Presentation of Data

Table 3.12.1. Means and standard deviations gradewise as well as sexwise for the various subsamples

S. No.	Grade	Sex	Mean	Standard deviation
1.	VI	Boys	5.70	8.06
		Girls	5.65	3.56
		Boys & Girls	5.67	12.46
2.	VII	Boys	10.85	6.49
		Girls	7.40	5.23
		Boys & Girls	9.13	12.20
3.	VIII	Boys	7.70	4.30
		Girls	7.10	5.38
		Boys & Girls	7.40	8.99
4.	IX	Boys	11.75	6.70
		Girls	7.55	3.52
		Boys & Girls	9.65	11.51
5.	X	Boys	12.8	5.49
		Girls	10.15	3.95
		Boys & Girls	11.48	10.42

Summary of Results

1. Grade means increase with age. Average performance favours boys rather than girls throughout the grades. It is also interesting to note that it takes four years for the grade means to double almost in grade X. Expectedly, the grade means on flexibility are less than the grade means on fluency, their values being 3.78, 4.08, 3.68, 4.05 and 5.25. Except a single fluctuation in grade VIII, grade means on flexibility also increase with age.

2. It is the privilege of teacher, largely speaking, to put questions to the ignorant students in the classroom whose answers he knows but his students in majority of the cases do not know. In this problem, we expected our pupils to raise those questions on cow and cycle (one living and other nonliving within their experience) whose answers they did not know. They were put in a new sort of thinking situation in which there was no ceiling on the number of questions to be formulated qualitatively as well as quantitatively. However, there was only one controlling factor, i.e., they were to write only those questions whose answers they did not know. In this context, it is interesting to point out that 26 per cent of the pupils coming from all grades VI to X were influenced more by the content rather than the form of the problem as shown in the following table.

Table 3.12.2. Incidence of content influence gradewise in terms of number of errors gradewise as well as sexwise

S. No.	Grade	Sex	Error	I.Q. range	Total Boys & Girls
1.	VI	Boys	—	—	2
		Girls	2	72-75	
2.	VII	Boys	12	70-120	22
		Girls	10	70-107	
3.	VIII	Boys	6	70-95	18
		Girls	12	70-117	
4.	IX	Boys	4	70-95	5
		Girls	1	70	
5.	X	Boys	—	—	5
		Girls	5	70-102	
			N	=	52
			N (boys)	=	22
			N (girls)	=	30

It is interesting to note that before they grasp the problem from the form angle, they undergo a hump in grade VII as shown by the curve in Fig. 8.

3. It was not the purpose of this investigation either to list questions (problems); quantify them in categories or to arrange the problematic questions in terms of higher cognitive processes bordering on taxonomy. To stress the main reason for including a problem of this type was to judge pupil sensitivity to self-generated problematic situations, a matter of individual judgement. This enabled us to obtain two distinct variables, namely, fluency and flexibility, except with this restriction that all those questions or problems whose answers were too obvious were rejected. On further reading the various responses, a few others were again rejected, for example, questions on objects other than cow and cycle; and essay type statements which were not questions at all. To illustrate:

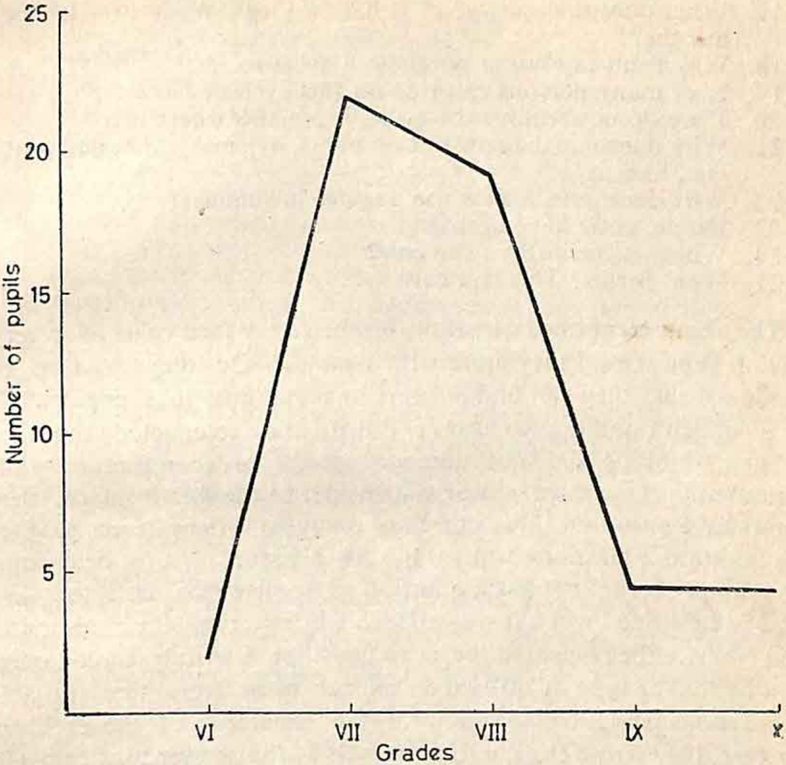


Fig. 8—Hump when content of the problem influences ability to formulate questions whose answers they do not know.

(A) *List of rejected questions*

1. Man rides on the cycle.
2. Cow eats grass or drinks water.
3. Cow is a domestic animal.
4. I like my cycle or love my cow. Can you tell me?
5. Cow (or cycle) is very useful to us.
6. Cycle contains air.
7. We get shoes from cow's skin.
8. We carry load on the cycle.
9. The cows or cycles are of many colours.
10. The cow gives young ones.
11. The cow eats grass. Cow does not eat bread like us.
12. The cycle has two wheels.
13. There are small hair on the body of the cow.
14. One remains under threat of fall while cycling.
15. Is cow a ball of fire? (A carry over question: Is sun a ball of fire.)

16. Why does not sun fall down on earth? (A carry over question.)
17. What does the cow give? It has two legs. What two (things) has she?
18. Why can't my father purchase a cycle for me?
19. How many persons can ride on this cycle?
20. If we do not remove the cycle, it remains where it is.
21. Why does not the cow put on pants, pyjamas, skirts and coats, etc., like us?
22. Why does not the cow use goggles in summer?
23. Do the cows have horns?
24. Where is the nose of the cow?
25. What is this? This is a cow.

The above mentioned questions on their very face value look very trivial. Why should they agitate their minds? Do they need to be reminded that they can find answers to these questions unaided? It is a difficult question to answer. But it is safe to conclude that they in their thinking did not undergo second order reflection while formulating these questions or statements. In the first instance, they could have answered these questions easily had they dared to face the question a bit more firmly. In the absence of this behaviour, they simply forgot the basic question to be answered. This tentative view is confirmed when it was noticed while reading their responses that they either repeated the same question in writing, took a question of similar type or utilised a helpful word from the available illustrations supplied to them in the statement of the problem. Further, they wrote the question as well as the answer to it below it. So far as the knowledge of the investigator goes, children do not undergo this experience in their day-do-day learning. It is, possibly speaking, due to their force of habit that each one of them; as soon as, read the question began to think of answering it. In this heat of the moment, he failed to resist the temptation and hence was led astray. Lastly, most of these questions came from pupils belonging to the grades of VI, VII and VIII (N=42) affecting 35 per cent of the pupils studying in these grades. After that, a sharp drop occurs in grades IX and X (N=10), the tendency persisting among 12.5 per cent of the pupils studying in these two grades.

(B) *List of questions accepted*

It is not possible due to the limited space available to list all acceptable questions. However, a few interesting ones for illustration purposes are mentioned below:

1. Why does the cycle move at all?
2. Why can't a cow talk like us?

3. Why do not the Hindus eat cow's meat?
4. Why does a cycle have two wheels or a cow four legs?
5. Why is there little fat in cow's milk?
6. The cow eats green grass but gives white milk. Why?
7. How does the cow from cud?
8. Why does not the cycle move when its tyres are deflated?
9. Why is a cow gifted to the Brahmins at the time of death?
10. Why can't a cycle move like the motor car or bus?
11. What is the significance of the cycle in the world?
12. What is the use of ball bearings?.
13. What is the average or normal temperature of a cow?
14. What is the maximum speed of the cycle?
15. Suppose the cow's horn is broken. Will it come up? How?
16. Can you count the hair on a cow's tail or body?
17. If the cycle can go on the road, why not up in the air?
18. Was the inventor of the cycle an engineer?
19. Had the cow four legs on its back, how would she have moved?
20. What is the maximum weight of the cow or the cycle?
21. The calf grows into an ox. How?
22. Who invented the cycle or the wheels of the cycle?

(C) *Types of questions asked*

Just to interrupt, puplis have asked all sorts of questions which could be classified into several categories, over 16 in number. This provided us a rough measure of flexibility, and an additional scheme of thought. Following categories were encountered in their responses:

1. What types: What, what else, for what and what happens.
2. Why?
3. How types: how; how many and how much
4. When
5. Which
6. Whether types: whether it happens or not and it is true or not.
7. Does and Do.
8. Is and Are.
9. If
10. Either or
11. Name and Describe
12. Give (the number of)
13. Can
14. Tell
15. Had there been
16. Miscellaneous. Combining two questions or writing a sentence or two ending in a question.

onsider other acceptable questions in continuation:

23. Man can cycle. Why cannot a cow?
24. Tell me the maximum age of the cow.
25. Which cow gives maximum milk? Why?
26. What gives whiteness to the cow's milk?

27. A cycle has two wheels. Can we have one with one wheel?
28. The cow has two horns. The donkey has none. Why?
29. The cycle has two wheels. Then what is the use of the chain connecting the two?
30. How much time is taken to manufacture a cycle?
31. How many bones has a cow?
32. How many components has a cycle?
33. For what different purposes can the cycle be used?
34. How is milk formed in the cow's body?
35. Are there any bacteria in cow's milk?
36. Would the cycle move had it been very heavy?
37. The cow's tail is visible and man's not. Why?
38. The cow loves her calf. Does the calf in turn also love her mother?
39. If cows refuse to give milk, will we then also worship them?
40. How does the cow breathe?
41. If a cow falls ill, will she give healthy milk?
42. Can you tell me the average weight of the cycle?
43. What are the factors considered for the manufacture of cycles? I hope there are some.
44. Is it possible to produce a cycle cheaply?
45. Name all the parts of the cow or the cycle.
46. Our teeth and cow's teeth differ. Why and in what ways?
47. Give the make of this cycle.
48. When do the cows usually get up in the morning?
49. What would have happened, had there been no cycles (or cows)?
50. Cycle has two wheels. The rickshaw has three. The scooter has two. The third is spare. Now can the cycle or rickshaw be changed into a scooter for getting good speed?

4. It is interesting to have a look not only at the relationships between fluency and flexibility but also between the two and the other outside variables. The correlation matrix for the composite sample (N=200) indicates that fluency and flexibility as defined in this study are very strongly correlated with each other (r=.6028). Considering usual levels of significance (5 per cent and 1 per cent); both are insignificantly related with: home adjustment; understanding the problem; felt difficulty of the problem; confidence in the problem;i nterest in the problem; counting rectangles maximally problem; seeing the problem as a whole, i.e., using four digits at a time; and stating the hypotheses. In addition, taking flexibility in isolation, it is further insignificantly correlated with health adjustment; social adjustment; emotional adjustment; school adjustment; positive summation series problem; hotel problem involving monetary transactions; and generalisation to algebraic symbols (summation). All the remaining correlations are significant between the two variables

on the one hand and the remaining outside variables on the other hand. It is intriguing to note why fluency and flexibility should be insignificantly related to seeing the problem as a whole as well as stating hypotheses. Is it due to the incomplete nature of the problem or has it been pitched at a higher cognitive level? It need not be ignored that both the variables are positively and significantly correlated with: stating procedures and proposing tests.

13: BEAKERS PROBLEM

The Problem

There are four beakers on the table. They are labelled A, B, C, and D as shown in the diagram given below. A small distance away, there is another beaker Z. One day, Ram did an experiment. Ram took parts of the contents from beakers A, B, C, D and put them in another empty beaker K. After this, he took some liquid from beaker Z and put it in the beaker K. An interesting thing happened. The entire liquid in K turned yellow. After sometime, his friend Mohan came and enquired of his friend about the formation of the yellow colour in the beaker Z. While answering this question, Ram forgot how he had done it. Now Ram carried out several experiments to obtain yellow colour in the beaker Z but failed. Your problem is to write down all the possible methods by which yellow colour could be obtained in the beaker Z. Please remember that you are to write down all the possible methods. Before you begin solving this question, please answer the following questions:

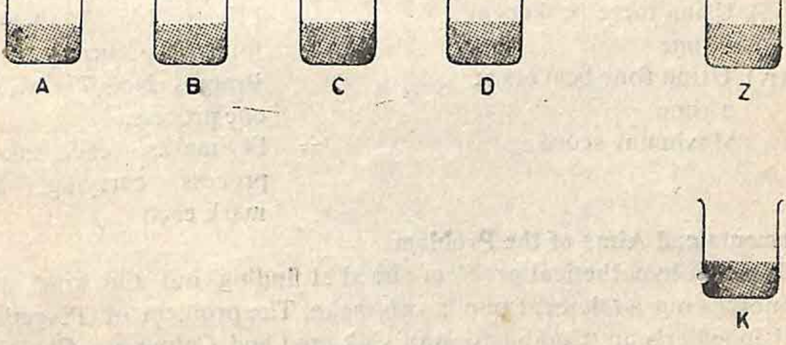


Fig. 9—Experimental setup.

(a) Have you done a problem of this type before? Yes/No
(b) Do you understand this problem? Yes/A bit/No
(c) Do you find this problem difficult? Yes/A bit/No
(d) Can you solve this problem? Yes/A bit/No
(e) Do you find any difficult word in this problem? Yes/No

If yes, write only those words below:

Now solve the above mentioned problem.

Method No. 1 First of all, Ram took out some material from beaker A and put it in beaker K. After this, he took some material from beaker Z and put it in beaker K. Yellow colour did not appear. He did another experiment.

Method No. 2

Method No. 3 and so on.

Manner of Presentation

The problem was presented as mentioned below. The arrangement of beakers: A, B, C, D, K, and Z was shown by placing six beakers on the table. The dropper was also shown for lifting material from beakers. Method No. 1 was fully explained as one of the possible solutions to the problem. If the pupil did not understand it, he was allowed to read it again. Again the step was explained. It was ensured fully that everyone understood method number 1.

Scoring

There are four processes involved in this problem which are relevant to the solution of the problem. These are:

(i) Using one beaker at a time	Process No. 74 with three sub-processes.
(ii) Using two beakers at a time	Process No. 75 with six sub-processescs.
(iii) Using three beakers at a time	Process No. 76 with four sub-processes.
(iv) Using four beakers at a time	Process No. 77 with one process.
Maximum score	= 14 marks, each sub-process carrying a mark each.

Elements and Aims of the Problem

This is a hypothetical problem aimed at finding out the kind of groupings our adolescent pupils can make. The problem of Piaget's and Inhelder's on 'Combination of Coloured and Colourless Chemical Bodies' is superior to the present problem for the latter was not

presented concretely [7]. The investigator tried this problem as a paper pencil test but failed because it became incomprehensible especially to pupils of grades VI to VIII. Therefore, on practical considerations, the problem as used by Piaget and Inhelder was modified by making it an absent situation but at, the same time, retaining its very essentials. In this restricted context, this problem was aimed at investigating the following:

(a) Up to what extent can the adolescent pupils structure essential requirements of the problem and attack it profitably?

(b) Up to what extent can they exhaust all possibilities while tackling a problem of this nature?

(c) What arbitrary errors do they make while tackling this problem?

Presentation of Data

Table 3.13.1. Means and standard deviations gradewise as well as sexwise for the various sub-samples

S. No.	Grade	Sex	Mean	Standard deviation
1.	VI	Boys	1.30	1.23
		Girls	1.55	1.36
		Boys & Girls	1.43	2.60
2.	VII	Boys	1.45	1.16
		Girls	1.20	.75
		Boys & Girls	1.33	1.97
3.	VIII	Boys	2.2	14.70
		Girls	.95	.87
		Boys & Girls	1.58	3.40
4.	IX	Boys	2.9	2.39
		Girls	2.8	1.70
		Boys & Girls	2.85	4.16
5.	X	Boys	4.8	3.23
		Girls	6.35	3.71
		Boys & Girls	5.58	7.13

Table 3.13.2. Acquisition of range of combinations gradewise as well as sex-wise in terms of number of pupils on exhausting the various possibilities relating to the beaker problem

Grade	Sex	0	1-3	4-6	7-9	10-12	13+
VI	Boys	7	13	—	—	—	—
	Girls	6	12	2	—	—	—
VII	Boys	1	17	2	—	—	—
	Girls	2	18	—	—	—	—
VIII	Boys	—	12	8	—	—	—
	Girls	5	14	1	—	—	—
IX	Boys	—	12	6	2	—	—
	Girls	2	8	10	—	—	—
X	Boys	—	2	16	—	1	1
	Girls	—	2	11	3	1	3
	Total	23	110	56	5	2	4

Table 3.13.3. Number of combinations given gradewise as well as sexwise on processes 74, 75, 76 and 77 of problem No. 13

Grade	Sex	Process No. 74	Process No. 75	Process No. 76	Process No 77
VI	Boys	14	7	0	5
	Girls	19	3	0	9
VII	Boys	7	3	2	17
	Girls	6	2	1	14
VIII	Boys	24	0	0	20
	Girls	4	0	0	15
IX	Boys	27	5	6	19
	Girls	27	11	6	16
X	Boys	51	18	11	20
	Girls	54	32	23	18
	Total	233	81	49	153

Table 3.13. 4. (Sub-Process Acquisition)—Number of combinations in percentages given gradewise as well as sexwise as processes 74, 75, 76 and 77 of problem No. 13

Grade	Sex	Sub-processive		Acquisition	
VI	Boys	23.3	5.83	0	25
	Girls	13.57	2.5	0	45
VII	Boys	11.66	2.5	2.5	85
	Girls	10	1.66	1.25	70
VIII	Boys	40	0	0	100
	Girls	6.66	0	0	75
IX	Boys	45	4.16	7.5	95
	Girls	45	9.16	7.5	80
X	Boys	85	15	13.75	100
	Girls	90	26.6	28.75	90
Overall percentage		38.3	6.25	6.13	76.5

Summary of Results

The main results on this problem indicate:

1. Except a small fluctuation in grade VII, grade means increase with age which is an expected finding. Average performance favours girls of grades VI and X whereas it favours boys of the remaining grades, i.e., VII, VIII and IX. The highest mean performance of grade X girls is less than half of the maximum score on this problem which clearly shows that most of the adolescent pupils are not in a position to exhaust all the possibilities inhering a combinatorial problem.

2. Contrary to Piaget, adolescent pupils are not in a position to exhaust all possibilities. Considering even a semi-maximum level of possibilities, it is only in Class IX and X that 40 per cent and 67.5 per cent of the pupils are in a position to do so. This ability to exhaust a half of the maximum possibilities exists little in grades VI, VII and VIII, the corresponding percentages being 0, 10 and 23.5. It is, therefore, least surprising that about one-third pupils of grade VI are not in a position to consider even a single possibility. In grade VIII, that is, two years later even 12.5 per cent girls of that grade are not in a position to suggest a single possibility. Considering the pooled sample (N=200), only 55 per cent of the pupils are in a

position to suggest only one two three combinations (possibilities) especially when one of the illustrations is provided to start their thinking on the problem (Table 3.13.2).

3. Considering that the fourteen distinct sub-processes of thought which when classified or categorised fall into four distinct categories: Taking one beaker at a time (3 sub-processes); Taking two beakers at a time (6 sub-processes); Taking three beakers at a time (4 sub-processes); and taking all the four beakers (1 sub-processes); there are 60, 120, 80 and 20 combinations (possibilities) involved in each grade sexwise (N=20 boys or girls to be multiplied with the number of sub-processes in each category). The resulting individual process-wise acquisition is shown in Table 3.13.4 which indicates:

(i) Taking one beaker at a time. It is only in grade X that almost all the pupils are in a position to exhaust all the combinations in that category. The percentage declines to 45 per cent in case of pupils of grade IX.

(ii) Taking two beakers at a time. Here, the overall performance is poor. It involves considering an extra beaker (B, C or D) along with the beaker already considered in (i). It is disturbing to point out that not a single boy or a girl of grade VIII could even suggest a single combination of this type which, theoretically speaking, allowed six different sub-processes under this process (S. No. 75). Even in grade X, the percentages of such combinations given by boys and girls are: 15 per cent and 26.6 per cent.

(iii) Taking three beakers at a time. Here, again the overall performance is poor. Not a single combination came from pupils of grades VI and VIII. It involves considering two extra beakers along with the beaker already considered in (i). In their own age groups, the percentage of such combinations by boys and girls of Grade IX and X are: 7.5 and 7.5 and 13.75 and 28.75 respectively.

(iv) Taking four beakers at a time. Only 20 combinations, sexwise were maximally possible in each grade. Here, it involved seeing the problem as a whole. Only 76.5 per cent combinations were given under this category. Except a single fluctuation in grade VII in the case of girls only, most of the combinations were given by pupils of grades VII to X. In grade VI, the percentages of combinations given by boys and girls are 25 and 45 which are low. This may be due to single aspect of the problem which was in fact short circuited on this problem.

4. It appears that the illustration appears to have assisted pupils in their thinking to exhaust most of the combinations failing under the category: using one beaker at a time. The overall percentages given

are: 42 per cent (B Z K); 37.5 per cent (D Z K); and 37 per cent (C Z K). The most difficult combinations to visualise were: BCDZK (5 per cent); ABDZK (5 per cent); ACZK (5 per cent); ACDZK (5.5 per cent); ADZK (5.5 per cent); BDZK (6 per cent); BCZK (6.5 per cent); and ABCZK, CDZK and ABZK (9 per cent each). This shows how difficult it is to restructure this problem from different aspects. This finding is further confirmed by the consideration that not a single pupil of grade VI, boy or girl, suggested the following combinations: ABZK, ACZK, ADZK, BCZK, BDZK, CDZK, ABCZK, ABDZK, ACDZK and BCDZK. Taking an overall view it is safe to conclude that the ability to attack the problem systematically has not developed among these pupils.

5. Large frequencies in the various cells reconfirm that a given combination or a given group of combinations is failed or suggested over a wide I.Q. range not only within a given grade but also across the various grades. The table below provides information about the number of combinations missed sexwise throughout the grades on processes 74, 75, 76 and 77, the maximum frequency of combinations on these processes being 60, 120, 80 and 20 respectively.

Table 3.13.5. Number of combinations missed gradewise as well as sexwise on processes, 74, 75, 76 and 77 of problem No. 13

Grade	Sex	Using one beaker at a time	Using two beakers	Using three beakers	Using four beakebs
VI	Boys	46	113	80	15
	Girls	41	117	80	11
VII	Boys	53	117	78	3
	Girls	54	118	79	6
VIII	Boys	36	120	80	0
	Girls	56	120	80	5
IX	Boys	33	115	74	1
	Girls	33	109	74	4
X	Boys	9	102	69	0
	Girls	6	88	57	2

Incidence or Arbitrary Errors

Failure to grasp the essence of the problem results in all sorts of errors. Then they bring several extraneous considerations into the

problematic situation which are in no way linked with the solution of the problem. They were too diverse and hence too difficult to count. Quite a few are mentioned below:

I. Took contents from Z and put into K.
II. At the last step, the yellow colour appeared.
III. How did the colour appear? He did the experiment.
IV. Repeating or reproducing the illustration or the question if the illustration fails.
V. Giving combinations like: ACK, BCDK, ABCDK, etc. The role of beaker Z was ignored. Alternatively, combination like ZK was given (K is an empty or collecting beaker only).
VI. Perform several experiments but nature of any experiment is not given. Alternatively, tried several methods and failed.
VII. Obtained contents from ABCD and boiled.
VIII. Drawing the diagram of a table and showing the positions of bottles on it.
IX. Give me an extra beaker. Then he forgot it.
X. (Comment) Either the beaker K did the trick or the mixing of four liquids.
XI. (Comment) I simply do not understand how contents in Z beaker change the colour.
XII. (Comment) It is not known how the yellow colour appeared in the first instance.
XIII. There is a suggestion or advice for Ram not to forget.
XIV. What is the business of the beaker K in the experiment?
XV. In fact, it is the duty of Ram to keep in his head the various experiments and tell them when asked.
XVI. I tell you the way. I am sorry it does not work now.
XVII. (Analogy and carry over effect of the illustration) in the second experiment, the colour did not appear because it did not appear in the first experiment.
XVIII. I will take the contents from the various bottles (beakers). I will try several combinations (but combinations are not given). If these do not work, then I will put yellow colour.
XIX. (Way out) I tell you the way. Empty the four beakers. Fill in them as usual. Then put some yellow colour.
XX. Transfer all the contents or liquids into the empty beaker K. Mix well. Be careful! Yellow colour will appear. If it does not, then put yellow water. Another response: cover it with yellow paper.
XXI. I tell you how the yellow colour can appear. I can guess that Z contains yellow colour. If not, then it is potassium iodine. You can taste it as well.
XXII. (See the incomplete response) In the first method the colour did not appear because Ram did not lift materials from beakers B, C, D and Z. He ought to have done this.
XXIII. There is yellow colour in the beaker Z which is not visible.
XXIV. (Combination not clear) Put materials from the remaining

beakers drop by drop into the beaker K. Then transfer this mixture drop by drop in the beaker Z. Something or somhewere the yellow colour must appear (what is your combination?). 'I do not know'.

XXV. Poor Ram had to perform several experiments. Why? I tell you. Take an extra beaker and put some water and yellow colour into it. Ram only forgot this. For this only, he had to perform several experiments.

XXVI. Take other beakers, say, D, E, F, G, H, then I, J, K, L; then M, N, O, P and so on. Put different materials and I hope the yellow colour will appear.

XXVII. Let Ram continue with this experiments. He should learn not to forget. I wish him 'good luck'!

XXVIII. I am dead sure that beaker K contains some yellow colour.

XXIX. I can say that these are all chemical liquids: water, acid, alkali and even scents.

XXX. Give me some litmus papers: red and blue.

XXXI. If my method does not work, I will take large and larger quantities.

XXXII. It is a terrible problem. I have to think.

XXXIII. It is a tricky problem because even I do not know when the colour will appear.

XXXIV. Ram ought to have labelled the beakers (which were already labelled).

XXXV. K beaker may be containing yellow colour (magic) or the beaker Z may contain yellow colour. I can also say that even the beaker K could be of yellow colour. Or the liquid turned yellow because there was little dye or colour on the palm of Ram which fell into beaker K during experimentation.

Discussion

Contrary to Piaget, it is strange to note that many adolescent pupils were led astray by the arbitrary errors committed in one form or another while tackling this problem. Even in grade X, the percentage of pupils so affected within that age group is as high as 57.5. There is another interesting observation. Throughout the grades, girls are affected more than boys as shown in the table given below:

	VI	VII	VIII	IX	X
Boys	65%	85%	60%	45%	50%
Girls	70%	85%	85%	85%	65%
Boys & Girls	67.5%	85%	72.5%	65%	57.5%

Further analysis of the data shows that the grade percentages as well as sexwise percentages on the incidence of arbitrary errors arising on account of the failure to accept the problem also suffer a hump each. One of such humps is shown in Fig. 10.

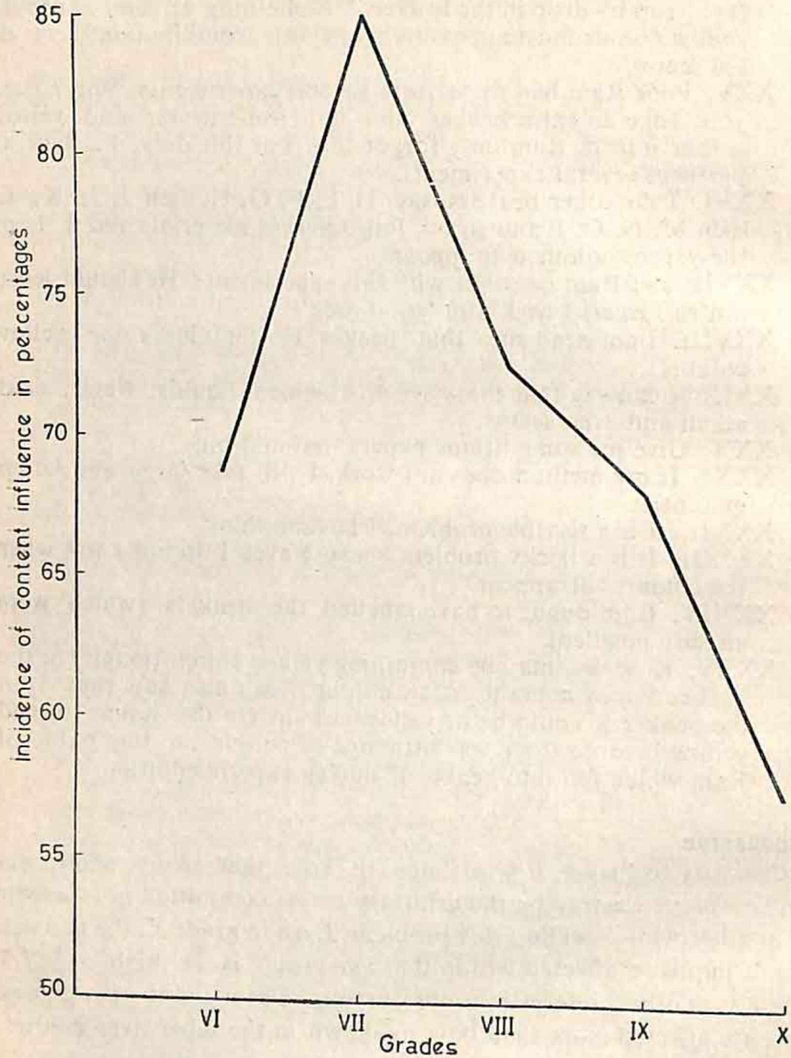


Fig. 10—Hump on the incidence of arbitrary errors arising due to the failure to accept the demand of the problem.

Several points of interest emerge. First, taking process No. 74, that is, taking one beaker at a time. Over 55 per cent of total combinations have been missed by the pupils of Grades VI to IX. Secondly, taking process No. 75, i.e., taking two beakers at a time. Over 73 per cent of the combinations have been missed by pupils of grades

VI to X. Thirdly, taking process No. 76, i.e., taking three beakers at a time. Over 71 per cent of the combinations have been missed by pupils of Grades VI to X. Fourthly, taking process No. 77, i.e., taking all the four beakers in any order. Only the boys of grade VIII and X have not missed any combination. All of them were thus able to see the problem as a whole. The percentages of those both sexwise and gradewise who missed seeing the problem as a whole are given whole is below:

	VI	VII	VIII	IX	X
Boys	75	15	0	5	0
Girls	55	30	25	20	5
Boys & Girls	65	22.5	12.5	12.5	2.5

Except in grade VI, it is more of girls rather than boys who have missed seeing the problem as a whole. Decline of percentages across the various grades leads us to conclude that seeing the problem as a whole is mastered gradually.

Sample Responses

Let us now mention below a few sample responses:

(i) G 5, I.Q. 80, Grade VI, Score 0
Ram was a small boy. Beakers: A, B, C and D. Took materials from beakers A, B, C and D and put these into beaker K. Beaker K is empty. The yellow colour appeared.

(ii) B 6, I.Q. 82, Grade VI, Score 0
Mohan inquired of Ram about the yellow colour. How did it appear? Ram carried out certain experiments, his bad luck that yellow did not appear.

(iii) G 6, I.Q. 82, Grade VI, Score 0
First of all, Ram poured water into a beaker. After a short while, he found that yellow colour did not appear. He tried again. He took materials from beakers A and C and put that stuff into beaker K. The yellow colour did not appear.

(iv) B 15, I.Q. 107, Grade VI, Score 0
You see that there are four beakers. In beaker K, put some yellow dye. Put material from beaker A or B or C or D into beaker K. You will always see yellow colour.

(v) G 18, I.Q. 115, Grade VI, Score 0
Took material from beaker B and put that into the empty beaker K. It won't work. Take another yellow beaker and put that K material into it. It will just look yellow.

(vi) G 69, I.Q. 92, Grade IX, Score 0
I will proceed as follows. Ram should take water in two or three beakers and add yellow dye into one of these. Then transfer a small amount of materials from beakers A, B, C, D each into the

beaker K. Then add water from the remaining beakers. Yellow colour will appear.

(vii) G 30, I.Q. 95, Grade VIII, Score 1
Take four bottles of water (chemicals). The colour turned yellow because all the four bottles stood in a row.
Took some materials from bottles A, B, C and D and put it into K. Then took out some material from beaker Z and put it into beaker K. The yellow colour may come.

(viii) G 38, I.Q. 115, Grade VII, Score 1
I tell you the way. Take eight glasses of different colours: Yellow, red, green, blue, whitish and golden etc. Put materials from Beakers A, B, C and D into these beakers. The materials in yellow definitely look yellow.
We have four boxes, you see.
Two of us (pointing to a friend) did the experiment. We put materials into these boxes. Yellow colour did not come.
Let us now take four beakers, A, B, C and D. Take material from each beaker and put it into beaker K. Then put some material from beaker Z into it. Now the yellow colour appeared.

(ix) B 41. Grade VIII, I.Q. 70, Score 1

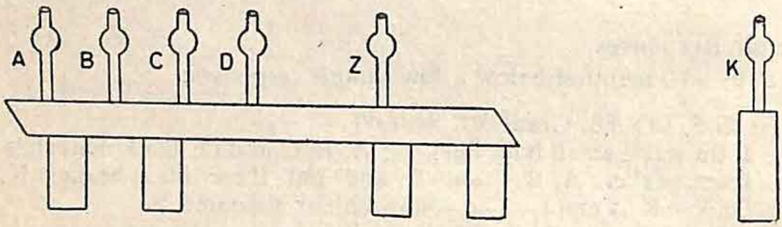


Fig. 11—One of the specimen responses pictorially.

I drew the diagram first. I have also shown the bottles. The empty bottle K is on a high stool.
I took materials from beakers, A, B, C and D one by one and put that into the empty beaker K. Then I took out some material from beaker Z and put that it into the beaker K. The colour turned yellow.

(x) B 20, I.Q. 120, Grade VI, Score 3
In a roundabout way suggests the following combination BKZ which is, in fact, BZK. There must be yellow colour in Z which is not visible to the eye. In C, it is whitish. I took a little colour from C, a bit more from Z and put both in the beaker K. Yellow colour should appear.

(xi) G 55, I.Q. 107, Grade X, Score 14.
Ram took material from beakers B and Z and put into beaker K. Yellow colour did not appear.
So he should take materials one by one from the beakers C and D. So the combinations are:

AZK, (already done)
BZK, CZK and DZK
The other combinations are :
ABKZ, ACKZ, ADKZ, BDKZ, CDKZ, BCKZ,
(taking two beakers at a time)
ABCKZ, ABDKZ, BCDKZ, ACZDK
(taking three beakers at a time)
Lastly, ABCDZK
(taking all the four beakers)
I think I have finished all the combinations. I do not see any other combination, still left.

What do they do when they fail to solve the problem? They complicate it by bringing several extraneous considerations into the situation in which they then carry out quite complicated or far fetched reasonings in another otherwise simple situation, which are simply not blind. Even when they carefully read the problem along with the illustration that gives scent to the solution of the problem, they continue to rub the problem at the wrong end. It is, therefore, least surprising that when failure to grasp the essence of the problem in the absence of the corresponding cognitive structure or scheme of thought occurs, all sorts of solutions arise which range from repeating, rephrasing or reproducing the beaker through obtaining an extra or several beakers using all the 26 alphabets to closing all the possibilities even once the understanding dawns that this particular solution must give the yellow colour. A few other helpful suggestions are also provided: be more careful in future, poor Ram should not have forgotten the experiment for he could well have written it, earlier, thus, avoiding trouble later on; emptying and re-emptying the various beakers; giving a few more chances to discover the solution; and committing the final experiment to memory. Apart from these, there are two other trends as well. First, why should 'yellow' does not ask or expect them to question just this part of the problem. Secondly, if they just accepted, the reason well could be some invisible colour in beaker Z (which could be a case of magic or jugglery) which appears in beaker K. If this is not, colour the K beaker yellow or wrap it in a yellow transparent paper. This solution amounts to using the basic information and concept underlying the science of colours. Our pupils just do not stop at this for they do further in their thinking, a distinguishing characteristic of their thought.

If the preceding methods have not worked, why not boil the chemicals? Why not use litmus papers or acids, alkalies, water and scents, etc.? These proposals amount to physical and chemical ex-

perimentation to a certain extent. This is usually done in the laboratory while investigating an unknown substance. In this context, their consequent thinking on this problem is quite respectable for no option even external to the problem is excluded from consideration to crack it up. Chemicals like drugs are labelled. Poison is clearly marked. Why? For quick distinction! Whereas the psychologist has its own limitations, any scoring system should not set aside such approaches. For example, if no illustration is given, beaker Z becomes beaker F and joins its earlier partners. The solution to the problem then becomes very difficult for it becomes an open problem in which every option has to be suggested. This very problem may then throw up several arbitrary errors making, in the process, any distinction between meaningful and arbitrary error difficult. The reason for this is that there are no external criteria against which a given option can be tested. This then necessitates either rephrasing the problem or imaginatively testing half the options with fewer and fewer control experiments. To cut short the discussion, our data abundantly indicated that the above mentioned arbitrary responses attempted at tackling the problem whether at the end solution emerged or not. This point is generally missed because solution to the problem is known to the psychologist. If the solution, unfortunately, is not known to the psychologist, distinction between relevant and irrelevant variable, in fact, ceases (the words easier said than explained) for they have to be shown so by the strength of general as well as (or) specific argument supported by observation or mathematics or both.

Lastly, let us see how this problem correlates with the other 44 variables. It correlates insignificantly at the conventional level of significance with the following outside variables: social adjustment; understanding the problem; felt difficulty of the problem and interest in the problem. With all the other variables, it correlates positively and significantly. There are a few interesting observations as well. First, it correlates more with grade than with intelligence (correlations being .5439 and .3534). Secondly, it correlates more significantly with some problems and schemes of thought than with intelligence. Some examples are: negative summation series problem, proportion problem, rectangular cube problem, combinatorial digital problem, questions involving wrong answers, nine dot problem, fish problem, spring balance problem, proposing tests problem, and using constant difference, using summation, using proportion, using three digits at a time, generalisation to algebraic symbolic (proportion); stating procedure and proposing tests.

14: FISH PROBLEM

The Problem

There is a rectangular fish tank made of thin glass. It is 52 cm deep. It is full of water and a fish is swimming in it. A boy looks down on the fish from the above surface and the fish appears to swim 24 cm below it. It is its apparent depth. His friend put his head under the tank at the same time and the fish appeared him to swim 12 cm away from the bottom. It is also its apparent depth. It is given to you that the ratio between the real depth and the apparent depth is 4:3. To put in other words, it means that if the fish is really 4 cms away from the above surface or from the bottom, it will appear to be only 3 cms from the top or bottom. You can ignore the thickness of the glass in your calculations. Your problem is to find the thickness of the fish.

Before you begin solving this problem, please answer the following six questions as stated in problem No. 13. Please solve this problem now:

First method

1. After reading, draw a diagram for the problem.
2. What is the real depth of the tank? =78 R
3. What is the apparent depth of the fish seen from the top? =79 R
4. What is the real depth of the fish when seen from the top? =80
5. What is the apparent depth of the fish when seen from the bottom? =81 R
6. What is the real depth of the fish when seen from the bottom? =82
7. What is the combined real depth(s) of the fish when seen from above as well as from below? =83
8. What is the real depth of the tank? =84 R
9. What is the thickness of the fish now? =85

Second method

1. What is the real depth of the tank? =86 R
2. What is the apparent depth of the tank? =87
3. What is the apparent depth of the fish when seen from above? =88 R
4. What is the apparent depth of the fish when seen from below? =89 R

5. What is the combined apparent depth(s) of the fish when seen from above as well as from below? =90
6. What is the apparent depth of the tank? (Repeat item) =91 R
7. What is the apparent thickness of the fish? =92
8. What is the real thickness of the fish? . =93
9. State your method of attack? =94

Manner of Presentation

The problem was presented as mentioned above. A small model based upon the essentials of this problem was shown. Except touching the fish, pupils were allowed to have a close look at the model. Differences between real depth and apparent depth was explained especially to pupils of grades VI and VII. The other pupils already knew the distinction. No other hint was given.

Scoring

Items relating to the reading aspects of the problem were not as usual scored. Each process carried a mark each except process No. 94 which related to stating the method of attack carrying two marks. Maximum score=10 marks.

Elements and Aims of the Problem

It is a very thought provoking problem which needs well developed schemes of thought relating to summation and proportion. Speaking concretely, it needs a firm grasp of the relationship that real deptl. divided by apparent depth equals 4:3. It was presented in two forms, i.e., proportion preceding summation; and summation following proportion in the second part of the problem. When seen in this context, this problem, in comparison to other problems included in this study, possesses the following distinguishing characteristics. First, like most of them, the arithmetic content involved is too small. Secondly, unlike them, it is presented in two forms which solve the same problem in two different ways. Thirdly, the various multiple or multifacet questions pinpoint pupil thinking on the various processes without supplying their answers. Fourthly, if able to solve the problem, it expects them to verbalise their methods of attack at the end of the problem. In this context, this problem is aimed at investigating the following:

(a) Up to what extent can adolescent pupils handle this problem successfully?

(b) What extraneous considerations do they bring into the problematic situation while tackling this problem?

(c) What errors do they made while solving the problem?

(d) Up to what extent can they verbalise or state their methods of attack?

Table 3.14.1. Means and standard deviations gradewise as well as sexwise for the various sub-samples

S. No.	Grade	Sex	Mean	Standard deviation
1.	VI	Boys	.05	.22
		Girls	.05	.22
		Boys & Girls	.05	.44
2.	VII	Boys	.25	.43
		Girls	.3	.46
		Boys & Girls	.28	.89
3.	VIII	Boys	1.6	2.73
		Girls	3.5	3.28
		Boys & Girls	2.56	6.32
4.	IX	Boys	3.55	2.77
		Girls	1.05	1.53
		Boys & Girls	2.30	5.12
5.	X	Boys	6.3	2.48
		Girls	3.5	3.38
		Boys & Girls	4.9	6.54

Table 3.14.2. Number of pupils gradewise as well as sexwise able to verbalise their method of attacking the problem partially

S. No.	Grade	Sex	0	1	2
1.	VI	Boys	20	—	—
		Girls	20	—	—
2.	VII	Boys	20	—	—
		Girls	20	—	—
3.	VIII	Boys	17	—	3
		Girls	11	3	6
4.	IX	Boys	14	1	5
		Girls	19	1	—
5.	X	Boys	7	—	13
		Girls	13	—	7

Summary of Results

The main results on this problem indicate:

1. The mean performance increases with grade. Average perfor-

mance favours girls in grades VII & VIII and boys in grades IX and X. Taking an overall view, the mean performance on this problem is poor in grades VI to VIII. It is also interesting to note that girls show a dip in their performance in grade IX, their grade means in grade VIII, IX and X being 3.5, 1.05 and 3.5 (see Table 3.14.1).

2. Secondly, a given problem, part of the problem or a process in that problem is solved or failed over a wide I.Q. range not only within the individual grades but also across the grades as well. As judged by large frequencies, this also applies to stating the methods of attack. For example, not a single pupil of grade VI and VII is in a position to state the methods of attack, reason being that they have miserably failed on this problem. In grade VIII, three boys (I.Q.'s ranging from 90-120) and six girls (I.Q.'s ranging from 80-120) are in a position to state their methods of attack firmly. Interestingly enough, girls draw a blank in the higher grade whereas the number of boys rose to 5, their I.Q.'s ranging from 72-115. In grade X, 13 boys and 7 girls with their I.Q.'s ranging quite widely are in a position to state their methods of attack.

3. Let us see how the problem is solved when presented in two ways: proportion preceding summation and summation preceding proportion by comparing their performance on processes: 85 and 93. The frequencies of pupils on these two process are shown below:

Table 3.14.3. Comparative performance in terms of number of pupils gradewise as well as sexwise along with consequent gain in percentage on processes relating to proportion preceding summation and summation preceding proportion of problem No. 14

S. No.	Grade	Sex	Process No. 85	Process No. 93	Gain in p.c.
1.	VI	Boys	0	0	—
		Girls	0	0	—
2.	VII	Boys	0	0	—
		Girls	0	0	—
3.	VIII	Boys	3	3	—
		Girls	6	9	15
4.	IX	Boys	5	6	5
		Girls	0	4	20
5.	X	Boys	13	18	25
		Girls	7	8	5

It appears that the manner of presentation did not help either way anyone of the pupils of grades VI and VII and only boys of grade VIII. Girls were assisted to the extent of 15 per cent in grade

VIII and boys to the extent of 5 per cent in grade IX. In grade IX, no girl was able to solve the problem using the scheme of proportion whereas 20 per cent were able to solve the second part of the problem using the scheme of summation. In grade X, extra 25 per cent boys and 5 per cent girls were able to solve this problem using the scheme of summation. The whole evidence indicates that when a problem is solvable through two schemes of thought, one inferior and the other superior, and if the latter is not well developed, the former may favour quite a few in solving the problem successfully. It is an hypothesis which needs to be verified in the future studies because of its educational import.

4. It correlates positively and significantly both with grade and intelligence, its correlation with grade being more than that of intelligence (r=.5649 and .16827). It correlates positively and significantly with all the problems and schemes of thought. It correlates insignificantly with the following variables: adjustment (home and social); and immediate test reactions to the problems on presentation (felt difficulty of the problem, confidence in the problem and interest in the problem).

Sample Responses

Let us consider a few sample responses which reflected the developing solution to this problem.

(i) G-1, I.Q. 70, Grade VI, Score 0
(How did you obtain your answer?)
You see the real depth of the tank is 52 cm (given). Thea pparent depth of the fish when seen from above is 24 cm (given).
The real depth of the fish when seen from above is 4 cm (Failure to apply the relationship or the given ratio 4 : 3).
The apparent depth of the fish is 12 cm which seen from the bottom (given).
The real depth of the fish when seen from below is 3 cm (Failure to apply the relationship or ratio 4 : 3 as in the earlier case).
The real depths of the fish when seen from above as well as from below ought to have been 4+3 but she put it as 52+4+3=69 (wrong computation)!
"It is too large", she persists. "There is some trouble with my calculations". The real depth of the tank is 4 cm (because 4 : 3 is the source of trouble). Hence the thickness of the fish is 4 cm (And this is greater than the depth of the tank)!
I do not know because I find that two depths are given: 52 cm and 4 cm. One of the two is only correct. The same reasoning continues in the second part of the problem except that the sum of the two apparent depths of the fish when seen from above and below is not 24+12 but 4+3=7, 4 and 3 are the ratio

figures. Finally, while playing with figures 3 and 4, she obtains the same thickness of the fish as she obtained in the first part of the problem. Answer is correct but not the reasoning for the implication of 4 : 3 relationship is not being considered at all.

(ii) G-11, I.Q. 97, Grade VI, Score 0.
The real depth of the tank is 52 cm (given). No, it ought to be $52 \times 3/4 = 39$ cm (contradicts the figure as given in the problem).
When seen from above, the real depth of the fish is 18 cm ($24 \times \frac{3}{4} = 18$). The apparent depth of the fish is 24 cm when seen from the top. She considers that the real depth of the fish when seen from the top ought to be less and hence multiplies it with $\frac{3}{4}$ (confusion between real depth and apparent depth).
It is precisely for this reason that the real depth of the fish when seen from the bottom comes out to be $12 \times \frac{3}{4} = 9$.
The real depth of the fish when seen from above and below ought to have been, according to her reasoning, $18 + 9 = 27$ but she reports it to be 12 cm (the apparent depth of the fish when seen from below plus the wrongly calculated real depth of the fish when seen from bottom ($12 + 9$).
She now reads the problem carefully and reports that the real depth of the tank is 52 cm (as given) but does not correct the same in case of the earlier computed test items seeking the same information. The real depth of the fish is 25 cm because $21 + 4$ cm (one of the ratio figures).
In the second part of the problem, she writes the apparent depths of the fish as 24 cm and 12 cm which are correct. But the apparent depth of the tank is 52 cm. There is confusion between real depth and apparent depth in her mind. The apparent thickness of the fish is 18 cm because $\frac{24+12}{2}$ (takes the average). Fails to calculate the real thickness of the fish.

(iii) B-29, I.Q. 92, Grade VIII, Score 1.
He draws a very good diagram, showing rectangular faces as well as distances. Then he responded as follows:
The problem is very tricky. If I can take the fish out, I can measure its thickness. It should be dead one. Otherwise it will slip. You see. (You need not take the fish out).
The real depth of the tank is 52 cm.
It (fish) seems 4 cm. Not so. It seems 3 cm and then it is 4 cm. Quite interesting, funny! The (seemed) apparent depth of the fish when seen from above is 24 cm. It is 12 cm when seen from the bottom up.
The thickness of the fish is 26 cm. (How?)
Because it is at the centre $\left(\frac{52}{2}\right)$.
(Is it at the centre in the problem?)

No. It can swim any where. I am wrong.
The real depth of the fish when seen from above is 25 cm.

(How?)

It is 24 cm+(4−3) because 1 cm is more than the apparent depth.
When seen from the bottom, its real depth is 12 cm +(4−3) =13 cm.
You see that if fish is 4 cm actual, it is 3 cm high when looked. (He fails to apply the information). Now the thickness of the fish is 52−(25+13)=14 cm. It is a very thick fish. In mathematics, it happens. In the second part of the problem, he reads the two apparent depths correctly, that is, 24+12.
How the thickness of the fish is 52−36=16 cm.
No.
It is (52−1)−36=15 cm.
It is its apparent thickness.
Its real thickness is 15 cm+1 cm =16 cm.
There is some difference because you are looking at the fish from top and bottom. And you see, the thickness of the fish is different. It tapers, you see, at the tail. The fish is very thick.

(iv) G-97, I.Q. 112, Grade X, Score 10.
The case under discussion is one of the most interesting cases encountered in this study. It indicates tentatively that if answer could be obtained without thinking, that is, by juggling with the figures, this mode of reasoning is generally resorted to provided it meets the teacher's approval. If not; and when placed in a tight testing situation in which supplying reason to the answer is the basic requirement, the possiblity exists of obtaining the answer within the structure as propounded by Professor Wertheimer. Otherwise a deceptively correct answer receives credit. Consider the case now.
She draws a diagram. She positions the fish, at different places for it swims.
The real depth of the tank is 52 cm.
The apparent thickness of the fish when seen from above is 28 cm because=24 (as given)+4 cm.
No, it is not 28 cm. I have done a mistake.
It is only 24 cm as is given in the problem.
The real depth of the fish when seen from above is 52 cm. It is because the fish is then at the bottom.

(Is it at the bottom in the question?)

No. It is also 12 cm away from the bottom. It looks that much away from the bottom. She rereads the question.
It has to be more than 24 cm so it is 24× 4/3=32 cm. It is 3 : 4 ratio, you see.

3 : 4
6 : 8
9 : 12
12 : 16

15 : 20
18 : 24
21 : 28
32 : 24

These quantities are maintained.
The apparent thickness of the fish when seen from the bottom is 12 cm. It is given.
The real thickness of the fish when it is seen from the bottom is 52 cm. The fish is at the top touching the glass.
No. It is 16 cm. You see against 12 above. The combined real depths are 52+52 if the fish is at the bottom as well as at the top. It is, so, 104 cm.
No. It is 32 cm+16 cm in this question.
It is (so) 48 cm.

The thickness of the fish is 2 cm because $\frac{52-48}{2}=2$

(Why 2 in the denominator?)

Because there are two depths.
No. I have taken care of this. It is 4 cm. (How?)
The depth of the tank is 52 cm. The real depths for the fish are 32 and 16 cm. Total is 48 cm.
No w 48+?=52 cm.
?= 4 cm.
In the second part, she calculates correctly the apparent thickness of the tank correctly: $39=52\times\frac{3}{4}$.
The apparent depth of the fish in the tank when seen from the top is 28 cm. (How?)
It is 28 because 24+4=28.
No. It is 24 because it is given.
The apparent depth of the fish when seen from the bottom is 16 cm. (How?)
It is 16 cm because 12+4=16.
No. It is 12 cm because it is given.
Let me read the problem again.
The combined apparent depth of the fish when seen from above and below are 52 cm +16 cm, i.e., 68 cm.
No, Sir. It is 24+12=36 cm.
The apparent depth of the tank is 40 cm. (How?)
It is 40 cm because 52−12=40.
Thinks!
The apparent depth of the tank is 39 cm. I have already found it. The figures, slip, you see.
So the apparent thickness of the fish is 3 cm because 39—36= 3 cm.
So the real thickness of the fish is 4 cm. (How?)
If apparent thickness is 3, the real thickness is 4.
Also, it is the same fish. It is the same problem put in two ways. The two thicknesses must agree. The only difference is that in the second part, you calculate the apparent depth first and then

the real depth or thickness, afterwards. In the first method, I was to calculate the real thickness first.

(How did you attack the problem?)

This problem puzzled me. The figures are interesting, with 24 and 12, you can both find $\frac{3}{4}$ or 4/3. They leave no remainders. So I made errors. I always felt that I was doing the things (calculations) the other way round. Then I thought and thought and the small questions helped me a lot in my thinking. So the method of solving this problem is simple. If you want to calculate the real depth from the apparent depth, multiply the figure with 4/3. (Why?)

Because the real depth is greater than the apparent depth in the ratio of 4 : 3. Add up the things. You arrive at the thickness of the fish.

But one has to be very careful in this question. Fish need not swim in this question for it is fixed for this question.

(v) B-91, I.Q. 97, Grade X, Score 10.

It is a puzzling question.

Can I take the fish out? (No)

Reads and rereads the question.

The real depth of the tank is 52 cm, you see.

The apparent thickness of the fish when seen from above is $24\times\frac{3}{4}=18$ cm. No. It is to be more.

So it is $24\times4/3=32$ cm.

The apparent thickness of the fish when seen from the bottom has to be $12\times4/3=16$ cm. (why?)

Because it has to be more.

So the total depth of the fish (means combined depths) is $32+16=48$ cm.

Now I must know something more.

Reads the question again.

Thinks !

Yes. The thickness of the fish has to be a small figure.

If the real depth is 4 cm; the apparent depth is 3 cm (it is given)

Now I have done it.

It has to be $52-48=4$ cm.

I have already done this in the above calculations.

The second part of the question appears to be the other way round.

I think the fish is the same (Yes. I have not changed the fish).

But it is swimming from point to point. Let it swim.

Its thickness has to stay the same.

Here, the apparent depths are given.

These are $24+12=36$.

So the real depth in all is $36\times4/3=48$.

Yes.

So the thickness of the fish is 4 cm$=52-48$.

If the tank is 4 cm, its apparent thickness is 3 cm.

So if the real thickness of the fish is 4 cm, its apparent thickness is 3 cm.

So the apparent thickness of the fish is 3 cm, you see.
(Yes, it is correct)
But tell me the apparent thickness of the tank.
It is not difficult, Sir. Thinks.
It is $39=52\times\frac{3}{4}$
I come to the same quantity.
You see $39=36+3$
$36+3=39$
Anyway, you calculate, the apparent thickness of the fish is $39-36$ cm$=3$ cm.
If it is 3 cm, the fish must look a bit more thicker, i.e. $3\times4/3=4$ cm.

(Could it be $3+1$?)

No. (Why?)
If the tank is 4 cm deep, it looks 3 cm.
If the tank is 8 cm deep, it looks 6 cm.
Here the difference is $6-4=2$ cm.
It is yes, also.
Suppose the tank is 4 cm deep, put a fish of 4 cm thickness into it. It has not to be very long. Then put water. From outside, it should look 3 cm. It is correct only then.
Can you give me your method? How did you solve it? These are two questions but, in fact, it is only one.
I will say as follows.
After spending some time on 3 : 4 or 4 : 3 relation, I discovered that it is not necessary to take the fish out. Even if you take it, it slips. It dies also (please do not bother). If I know the apparent depth, I then calculate the real depth. And this has to be more. So I multiplied both 24 and 12 with 4/3.
Total is equal to $32+16=48$.
Now, the tank is deeper than this.
And it is 52 cm. So I took away 48 from 52.
It is 4 cm.
Then I added the two apparent depths of the fish $(24+12)=36$.
I calculated the real depths$=36\times4/3=48$ cm.
So again, I took away 48 from 52. You see, I was doing the first question in the second part of the problem. Then it struck me because there was a question: What is the apparent depth of the tank? It has to be less. And it is $52\times\frac{3}{4}=39$. Then I calculated the apparent thickness of the fish $=39-36=3$ cm.
Again, I came to the same relation, if one is three, the other is four. If one is four, the other is three. I am a fool that I solved the second part of the problem by the first part (meant the first part of the problem).

(vi) G 160, I.Q 120, Grade X, Score 10.
Except a few changes in words, she follows more or less the same reasoning as manifested by case No. V. Consider the statement of her method of attack.
I never imagined that the problem could be done like this. But

as I read the various questions, it struck me that 3 : 4 or 4 : 3 has something to do with the solution. So I thought as follows: I found the real depths in the first part of the problem. Their total was 48 cm. I took this away from 52 because 48 + something (thickness) must equal to 52. I did not bother about the thickness of the fish as it was given. So 4 cm is the thickness of the fish.

This gave me the confidence to solve the second part of the problem. But I had my doubts because the fish was becoming small and so was the tank. I then turned the figures upside down. I had to. Because real depth is greater than apparent depth.

These sample responses indicate the varied ways to the solution of the problem. While solving the problems, the adolescent pupils do pass kind as well as cutting remarks on the problem. Examples are:

(i) Why should it be necessary to determine the thickness of the fish this way? There are simpler methods of doing it as well.
(ii) The weight of the fish is more important to find than its thickness.
(iii) It is a fine aquarium. Why not slip 2-3 fishes into it ?
(iv) I find this problem tricky and funny.
(v) Whereas I was trying to understand the problem correctly, I was making the calculations the other way round.
(vi) I want to know how the fish slipped into the tank.
(vii) Can you tell me the length and width of the tank? I want to know how large is the tank?
(viii) It is difficult to find the thickness of the fish because it is moving all the time.
(ix) Let me take the fish out of the tank and measure its thickness against a ruler. It may slip and then will die.
(x) I am a fool that I solved the second part of the problem by the first part (meant the first part of the problem).

Discussion

Then there is confusion between real depth and apparent depth. Even when clear, the real depth does not exceed apparent depth by a cm $(4-3)$ because they are connected by a proportional relationship. Hence a difference of 1 cm favouring real depth cannot be added to the apparent depths of the fish when seen from above or below. Precisely for this reason the thickness of the fish is not 26 cm $\left(\frac{52}{2}\right)$ because it is at the centre of 18 cm $\left(\frac{24+12}{2}\right)$, the last two being the apparent depths of the fish when seen from top and below. Two (2) appears in the denominator because either two things are involved (real depth and apparent depth) or fish is just at the centre or the middle point. Lastly, even when adolescent pupils are nearer to the solution of the problem, they play with

S. No.	Process No.	Errors committed	No. of errors	Dominant errors	Grade	
2	82	9, 10 or about 10, 12, 4, 20, 24, 21, 4:3 or 3:4, 3, 13=12+1, 36, 1, 40, 6, 0, 28, 120, 42, 14, 12+4×3, 52, 32, 84, 3:7, 76, 2, 36, 82, 22, 16, 26	31	4	VI VII VIII IX X	12 13 5 15 3
				12	VI VII VIII IX X	5 15 4 4 2
3.	83	15, 70, 12, 49, 76, 87, 4/3+3/4,3, 36=24+12, 25, 66, 7, 104, 24=24—3 14, 16, 3, 4, 120, 28, 43, 60, 6, 48, 50, 7, 9, 27, 68=40+28, 56, 26, 69, 24, 8, 125, 64, 46, 52, 184, 45, 46, 32, 10, 38	44	36 = 24+12	VI VII VIII IX X	5 10 8 17 2
4.	85	6, 18, 2, 12, 40, 15, 68, 8, 39=52×3/4, 3, 20, 14, 3/4 or 3:4, 52/4=13, $\frac{24+12}{2}=13$, 9, 16=52—36, 1, 9, 24, 28, 12/3=4, 36, 48, 70, 21, 43, 17, 25, 7, Can't say, 64, 32, 64, 82, 14, 52=76—24, 0, 10, $26=\frac{52}{2}$	38			
5.	[illegible]7	24, 3, 12, 52, 3:4 or 4:3 36, 32, 4, 8, 26, 0, 7, 64, 51, 32, 52, 49, 5, 1, 36, 68, 13, 46, 48, cm, Can't say, 2	27	24	VI VII VIII IX X	6 9 5 2 2

certain figures with a view to throw answers acceptable to the teacher by short-circuiting the processes of thinking underlying the various test items: and if accepted, they get extra credit without having really thought out the various answers. To put in other words, it amounts to get credit for the correct answer by using wrong processes of thought. They then fail to deduce the right answers from the conditional statements surrounding a problem containing a continuous chain of reasoning. They come out with the right answers only when they are confronted with their varied answers. In this context, contrary to Piaget, they do not resort to deducing answers scientifically from the problems but, instead, quickly feel the answer, respond successfully through guessing and get credit, in turn, for giving the correct answer using wrong processes of thought. Why? Because the final answer to the problem depends upon the following:

(i) The position of the fish is fixed momentarily and has to be reckoned the same throughout the solution of the problem.

(ii) Real depth is greater than the apparent depth holding a proportional relationship.

(iii) Ignoring the thickness of the glass, the real depths of the fish when seen from above and below plus the real thickness of the fish equal the real depth of the tank. The same relationship holds if real depths are replaced by apparent depths. The reciprocal relation as stated in (ii), enables to determine the real thickness of the fish.

Analysis of Errors

Failure to see or grasp the basic essentials of the problem results in all sorts of errors as shown in the table below.

Table 3.14.4. Errors committed, number of errors committed along with the distribution of dominant error(s) gradewise for the various sub-samples

S. No.	Process No.	Errors committed	No. of errors	Dominant errors	Grade	
1.	80	4, 24, 52, 2, 18, 40, 0, 23, 12, 8, 4:3 or 3:3, 96/3=32, 1, 6, 3, 25=24+1, 28, 9, 16, 26	20	24	VI	8
					VII	9
					VIII	13
					IX	11
					X	2
				4	VI	5
					VII	7
					VIII	4
					IX	11
					X	2

S. No.	Process No.	Errors committed	No. of errors	Dominant errors	Grade
6.	90	7, 52, 4:3 or 3:4, 11 cm, $\frac{3}{4}+\frac{4}{3}$, 53, 12+3=15, 31, 33, 3, 4, 12, 24, 6, 48, 16, 36, 86, 2, 43, 44, 73, 4, 5, Can't say, 134, 18, 9, 120, 27, 15, 7:7, 92, 10, 5, 28, 30, 68	37		
7.	92	52, 43, 32, 24, 12, 4, 3, 18, 7, Can't say, 2, $52\times6=312$, 36, 9, 77, 6, 70, 32, $\frac{52-12}{2}=\frac{40}{2}=20$, 82, 13, 48, 15, 8, 25, 69, 39, 1 cm, 2 cm, 40 cm, $\frac{15}{24}=\frac{5}{8}$, 28, 16.	33		
8.	93	4, AE, $4\times3=12$, 2, 24, 6, 12, 5, 29, 8, 5/8, 3 or 3:4 or 4:3, 52, 40, 72, 21, 200, Can't say, 9, 7, 36, 5, 4, 96, 26, 13, 48, 78, 108, 16, 14, 32, 23	32		

Pupils have to show mastery on the eight processes of thinking before they can fully solve the problem. In their failure to do so, they show highly individualistic modes of thought, making any common pattern of thinking least discernible. Simple test items have attracted larger number of errors much to the amazement of any worker seriously interested in investigating adolescent thought. This condensed information is presented in the table given below:

Table 3.14.5. Summary of number of errors along with the number of dominant errors which appeared on the various processes of Problem 14

S. No.	Processes of thinking	Process No.	No. of errors	Dominant errors
1.	What is the real depth of the fish when seen from the top?	80	20	2
2.	What is the real depth of the fish when seen from the bottom?	82	31	2
3.	What is the combined real depths of the fish when seen from above as well as from below?	83	44	1
4.	What is the thickness of the fish now?	85	39	—
5.	What is the apparent depth of the tank?	87	27	1
6.	What is the combined apparent depths of the fish when seen from above as well as from below?	90	37	—
7.	What is the apparent thickness of the fish?	92	33	—
8.	What is the real thickness of the fish?	93	32	—

The number of errors have ranged from 20 to 44. This clearly shows that adolescent pupils, each taken individually, consider any virtual relationship. When aggregated, they in a group situation appear to exhaust all possible (such) relationships, a problem of substantial educational significance. Pupils have to show mastery on the eight processes of thinking before they can fully solve the problem. Out of the eight processes of thinking under study, four do not show any dominant errors at all, two one each and the remaining two, two each. All these dominant errors suffer a hump each. One of such humps is shown for one of the dominant errors on process No. 83.

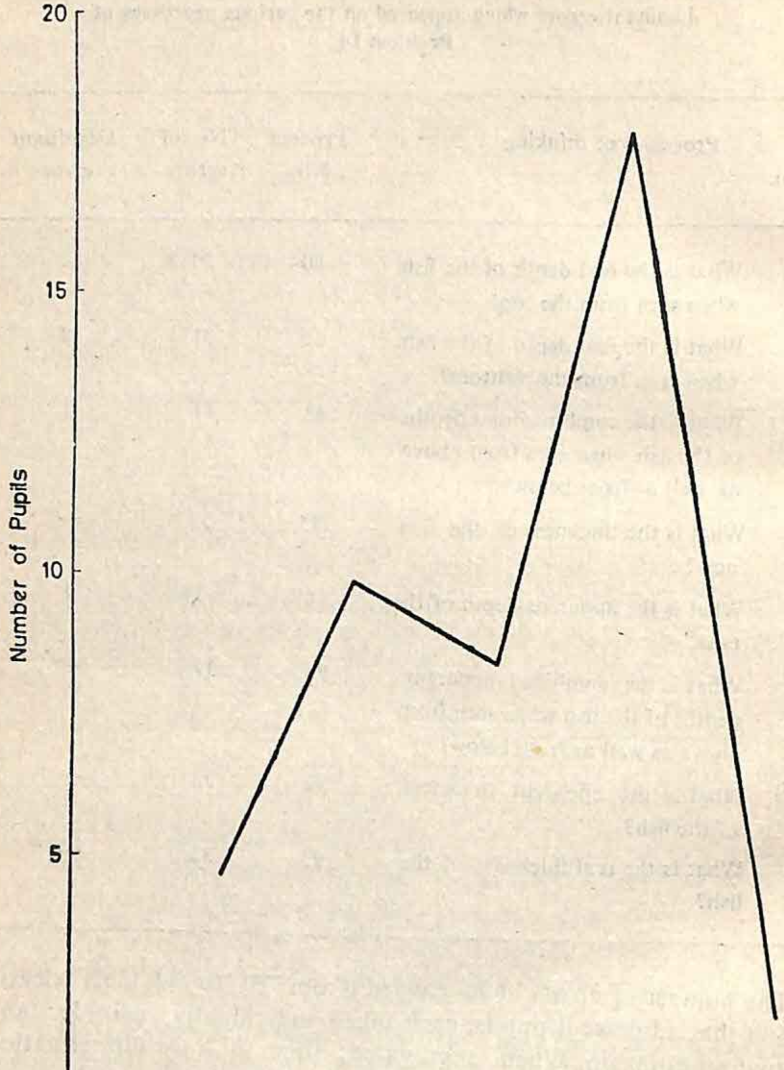


Fig. 12. A bihump of the dominant error on Process No. 83.

It may be added that quite a few individual errors which have not acquired the status of dominant errors on the basis of our criteria

also undergo a hump each. But those frequencies are too small to consider.

Educational Suggestion

It is possible to lead the pupils towards the solution of a problem by a series of well designed multifacet questions. Performance on this problem would have been much poorer (if not negligible) if this had not been done. It indicates that childern can be helped to structure problems but among these, the only ones who succeed, are those who have the basic schema or concepts needed to solve the problem available to them. It is only then that the concept of 'Gestalt', "whole" and the "Figure and Ground" are brought under control and the concept of 'insight' is made to function without the children knowing that they have actually solved the problem. They go on answering the problem bit by bit, without showing any apparent concern and when stuck, start wondering. What to do further? They then continue to fill in the blanks which they can fill in straightaway by reading from the problem. If they continue to persist, the second attack on the problem itemwise gets more concentrated. There is some feeling of relief and brightness on the face when they have computed the last missing item. This was our experience on this problem when it was administered.

15: SPRING BALANCE PROBLEM

The Problem

There is a spring balance with you. It has no markings on it. One day, your mother asked you to get her 15 grams of sugar. She wanted to test your ingenuity. For solving this problem, she gives you several beakers, each weighing 5 grams. Moreover, she also gives you a scale (a ruler), a pencil, thread and three weights of 1 gm, 2 gms, and 3 gms. You can tell me what else you need. That will be given. You have to weigh 15 gms of sugar in one attempt only. How will you solve this problem for your mother?

Before you begin solving this problem, please answer the following questions:

(a) Have you done a problem of this type? Yes/No
(b) Do you understand this problem? Yes/A Bit/No
(c) Do you find this problem difficult? Yes/A Bit/No
(d) Can you solve this problem? Yes/A Bit/No

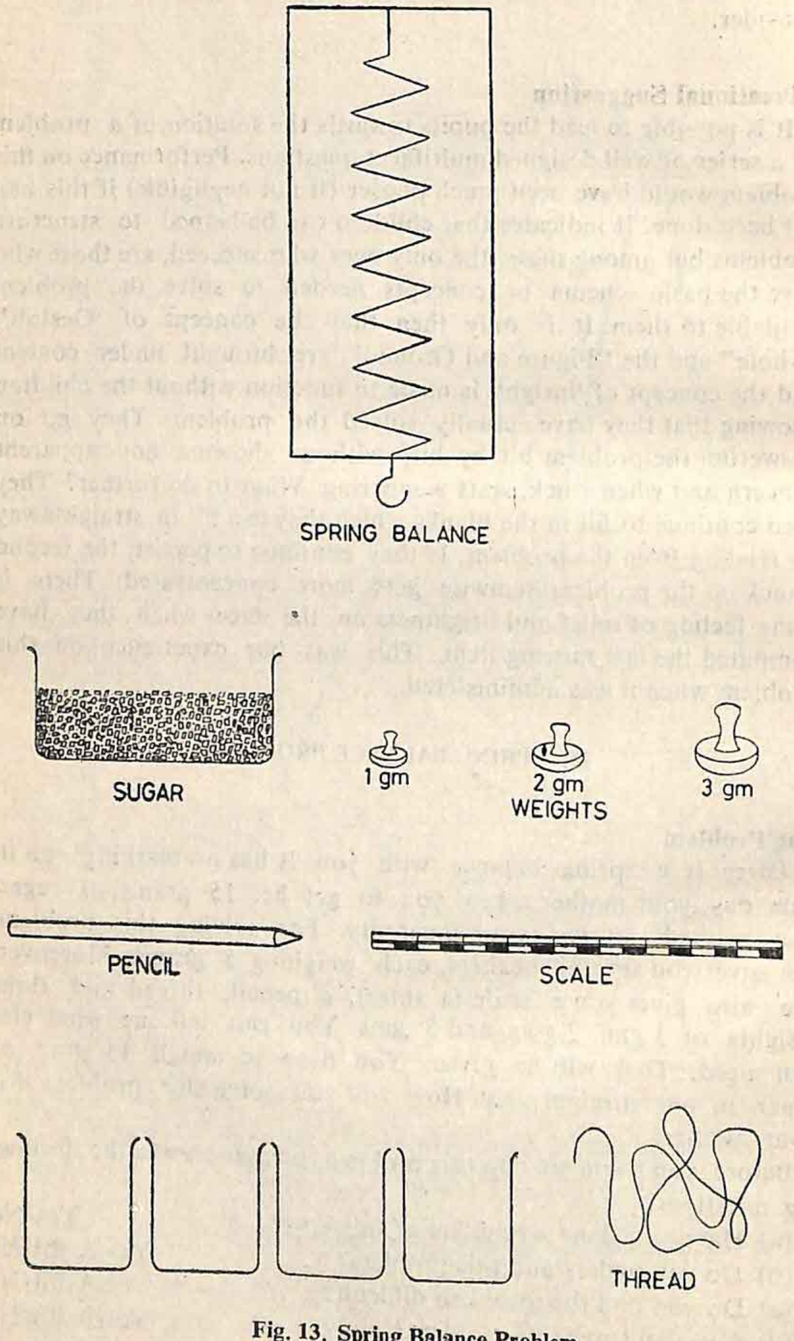


Fig. 13. Spring Balance Problem.

(e) Do you find any difficult word in this problem? Yes/No
If yes, write only those words.

Manner of Presentation

The problem was presented as mentioned above. Experimental materials, piece by piece, were shown. Each one of them was allowed to handle the material in any manner he or she liked. For example, the most popular act noticed here was the stretching of the spring balance.

Scoring

The following processes are involved in this problem.

1. Using beakers as weights	Process No. 95	1 mark
2. Using weights	Process No. 96	1 mark
3. Using beakers and weights	Process No. 97	1 mark
4. Discovering that extension is proportional to the stretching force	Process No. 98	1 mark
5. Suggesting calibration	Process No. 99	2 marks
6 Making the scale	Process No. 100	—
7. Stating the procedure	Process No. 101(a)	1 mark
8. Taking into account the weight of the beaker	Process No. 101(b)	1 mark
	Maximum score	8 mark

Elements and Aims of the Problem

This is a very simple, interesting and challenging problem which is usually encountered in the physics laboratory. This problem can be presented in various ways like presenting the half calibrated spring, or presenting it with some other pieces of apparatuses like pulleys, inclined plane and other specific gravity or density experiments. But on actual preadministration, the following form turned out to be quite suitabte for investigating the scheme of proportion. When the problem is read, it is apparent that it involves grouping suitable weights in three different ways with this difference that the weight of the beaker must be taken into account while measuring exactly the fifteen grams of sugar as demanded by the problem. In the second part of the problem, the same amount of sugar is to be weighed on an unmarked or uncalibrated spring balance. For this purpose, a ruler, a pencil and any two weights (making less than

15 gms) are given. Really speaking, one weight is also sufficient to solve this problem provided the pupil assumes that the extension of the spring balance is faithful, i.e., proportional to the stretching force. If not, he can then verify it with the help of any two weights. This was, however, not questioned by any of our pupils. So in the light of what has been said, this problem is aimed at investigating the following:

(a) What alternative methods do the adolescent pupils suggest while tackling this problem?

(b) Up to what extent can they suggest the calibration of the spring balance upon the principle that extension of the spring is proportional to the stretching force?

(c) Up to what extent can they take into account the weight of the beaker, i.e., the container?

Presentation of Data

Table 3.15.1. Means and standard deviations gradewise as well as sexwise for the various sub-samples

S. No.	Grade	Sex	Mean	Standard deviation
1.	VI	Boys	.15	.36
		Girls	.45	.45
		Boys & Girls	.30	.92
2.	VII	Boys	1.4	.49
		Girls	1.4	.49
		Boys & Girls	1.4	.98
3.	VIII	Boys	2.0	0.00
		Girls	2.25	1.09
		Boys & Girls	2.13	1.57
4.	IX	Boys	3.7	1.93
		Girls	2.65	1.39
		Boys & Girls	3.18	3.52
5.	X	Boys	7.25	4.60
		Girls	5.75	2.23
		Boys & Girls	6.50	2.49

Summary of Results

The main results on this problem indicate:

1. Grade means increase with age. Average performance favours girls in grades VI to VII and boys in grades IX and X.

2. Whereas 84.5 per cent of the pupils consider to use beakers as weights, only 28 per cent considered using given weights as weight in the solution of this problem. The possible explanation for this is that the given weights were not sufficient to solve the problem unless repeated weighings were taken. However, 67 per cent of the pupils considered using both weights and beakers in its solution.

3. The scheme of proportion inhering this problem is hardly developed among pupils of grades VI to VIII. In grade IX, it is just developed in 10 per cent of the pupils which rises to 65 per cent in grade X pupils. It is in grade X that they are in a position to suggest calibration and verbalise their methods of attack.

4. Pupils of grades VI to VIII are not in a position to consider the separation between the demand of the problem and the weight of the beaker. In grades IX and X, only 45 and 67.5 per cent of the pupils are able to do so.

5. As judged by the large (or small) frequencies in the various cells, it is safe to conclude that processes underlying this problem are solved successfully or failed over a wide I.Q. range, not only within the individual grades but also across the various grades.

6. The adolescent pupils commit a large number of arbitrary errors when they fail to grasp the essence of the problem. When they are added up, not only the two dominant errors but also all the arbitrary errors undergo a hump in grade VII.

7. Lastly, let us see how this problem correlates with other variables. It correlates more with grade than with intelligence ($r=.7964$ and $.2049$). It correlates significantly with all the problems as well as the schemes of thought. Next to grade, for example, it correlates very significantly with the following two schemes of thought: using summation and using proportion ($r=.7751$ and $.7370$). Its correlations with other variables like Generalisation (taken separately); and using insight are: .2855, .6344 and .5793. It is insignificantly correlated with the following immediate test reactions on presentation: understanding the problem, felt difficulty of the problem and interest in the problem.

16: PROPOSING TESTS PROBLEM

The Problem

One day, your science teacher gave you three small sticks exactly similar in appearance in all aspects. These were made, for your information, of iron, wood and sugar. Your problem is to find out

which stick is made of which material. Suggest as many possible experiments and tests to solve this problem as you can.

Presentation

Three pieces of chalks were laid on the table. They were said to be made of iron, wood and sugar (sweet). They were free to tackle this problem in any way they liked.

Scoring

One mark for each acceptable test or experiment.

Elements and Aims of the Problem

It is a simple problem involving proposing tests or experiments which could distinguish among three sticks made of iron, wood and sugar. Other problems on distinguishing among acids, fibres and materials of daily use were considered but were rejected because they did require certain amount of background chemical knowledge which pupils may not possess in equal measure throughout the grades. A defused problem in this context which could evoke variety of thought was, however, acceptable. Hence the choice of this problem lay for another reason as well, i.e. no physical and chemical tests were to be physically applied while proposing tests. They could easily, on the other hand, draw upon their everyday experiences or common knowledge about these objects. Lastly, this simple problem would have wide appeal to our pupils, provoking in the process, variety of thought. So this problem is aimed at investigating the following:

(a) Up to what extent can the adolescent pupils propose tests or experiments on three objects of daily experience which exactly look alike?

(b) Up to what extent pupils bring about extraneous considerations into the problem situation while proposing tests and experiments?

(c) What is the overall incidence of the various tests proposed in the succeeding grades?

Presentation of Data

Table 3.16.1. Means and standard deviations gradewise as well as sexwise for the various sub-samples

S. No.	Grade	Sex	Means	Standard deviation
1.	VI	Boys	1.7	.95
		Girls	.50	.81
		Boys & Girls	1.10	2.14
2.	VII	Boys	1.95	.80
		Girls	2.15	1.11
		Boys & Girls	2.05	1.99
3.	VIII	Boys	2.55	1.16
		Girls	3.45	1.12
		Boys & Girls	3.00	2.45
4.	IX	Boys	4.4	1.56
		Girls	3.25	1.44
		Boys & Girls	3.83	3.22
5.	X	Boys	6.15	2.93
		Girls	4.65	2.55
		Boys & Girls	5.4	5.7

Table 3.16.2. Number of tests proposed in terms of number of pupils gradewise as well as sexwise

S. No.	Grade	Sex	0	1	2	3	4	5	6	7	8	9	10	11	12
1.	VI	Boys	—	11	6	1	2	—	—	—	—	—	—	—	—
		Girls	14	2	4	—	—	—	—	—	—	—	—	—	—
2.	VII	Boys	1	4	14	4	—	—	—	—	—	—	—	—	—
		Girls	—	7	7	2	4	—	—	—	—	—	—	—	—
3.	VIII	Boys	—	3	8	6	2	—	1	—	—	—	—	—	—
		Girls	—	—	3	10	4	1	2	—	—	—	—	—	—
4.	IX	Boys	—	—	—	9	3	2	4	1	1	—	—	—	—
		Girls	—	2	5	4	6	2	—	1	—	—	—	—	—
5.	X	Boys	—	—	1	2	2	2	1	6	3	1	1	1	—
		Girls	—	—	5	3	3	2	4	1	—	1	—	—	1

Summary of Results

The main results on this problem indicated:

1. Average performance increases with grade. It favours boys of grades VI, IX and X and girls of grades VII and VIII. Both try hard to equalise their performance (see Table 3.16.1).

2. It is a telling comment on our educational system that 70 per cent of the girls belonging to grade VI could not propose a single test which could distinguish among three sticks made of iron, wood and sugar. The I.Q.'s ranged from 75 to 120. Secondly, the maximum number of tests proposed is 12. It is only in grades IX and X that 22.5 per cent of the pupils studying in these grades have been in a position to suggest more than six tests. And this percentage is too low when others have also put in more than eight years stay at school. The outstanding finding here is that majority of pupils do not care to exhaust all the possibilities while proposing tests. If anyone of them gets at one, he considers the second test as unnecessary for the problem any way stands solved. If compelled or placed in a tight situation, he may end up with another test. If so, the third stick is automatically fixed partially or fully by the process of elimination.

3. Let us now consider the popularity of the various tests proposed by the entire sample gradewise and sexwise. The various tests in order of popularity are: tasting, burning, breaking, judging weight, use of water, by striking, use of magnet, by heating, by cutting with knife, rusting, dropping from height, rubbing, use of acid, passing electric current and by suspending weight. Tasting comes at the top because the sweet stick tempts most of them except girls of grade VI (N=1). It should not be lost sight of that tasting chemicals even when known is not allowed in our schools. If this, test is banned, burning test comes second because in scientific experimentation, the spirit lamp comes handy in our day-to-day science teaching. The problem being open with no restriction imposed even that of finance, it is strange why the use of water should occupy the fifth position when all children know that wood floats on water and the iron goes down to the bottom. There is another interesting observation to make. When compelled to think hard, 7.5 per cent of the students have proposed the rusting test. They mostly belong to grades IX and X. It means they can await pronouncement of judgement even for a couple of days if experimentation so requires. It is quite creditable that they just considered this test.

The table also indicates some other depressing observations which do need urgent attention. No pupil of grade VI and no girl from

grades VI, VIII and X could propose the use of magnet as one of the tests to slove this problem. Not a single pupil from grades VI to VIII could suggest the heating test. The frequency rose to 1 each in grade IX for boys and girls. Only two boys of grade X could suggest this test. Not a single pupil from grades VI to IX could propose the acid test. The three per cent who did (came in equal numbers from grade X) did not specify it. Not a single pupil from grades VI to IX and a girl from grade X could imagine passing the electric current as one of the tests for solving this problem. These are very disturbing observations for pupils under study are making little use of their textbooks knowledge when teaching of General Science is fairly established on paper for the last twenty years.

4. In the lighter vein, it is generally said that the most efficient person is one who hardly does any work (input approaching zero). Similarly, the adolescent intellectual appears to be very efficient when he skips the questions. It then becomes very difficult to draw inferences about his intellectual behaviour for he, despite his potentiality, fails to float any tests or hypotheses. When this option is denied for the experimenter is still insisting on, he just cares to understand the problem. Once he does just that or begins to accept or understand the problem, he starts committing errors or mistakes which he could have avoided otherwise. Being at the transitory stage or perhaps in that stage of development, his thinking, as if sitting on the horns of a dilemma, begins to become erratic, undergoes a hump and disappears with increasing grades. Consider the following sample responses.

I. Let me draw the diagram. I think it is not necessary in answering this question.

II. In science, we make measurements. Let me measure its length and thickness, etc. He then describes the question or rewords the question.

III. Wood is used for burning. Sugar is white and used in tea. One can add it to milk. Sugar is sweet, you see.

IV. I guess or declare that the second stick is iron. How can iron be white? Alternatively, I can judge wood simply by seeing.

V. Let me add milk here. Milk and sugar go together (on reflection, it can be one of the acceptable proposed tests for sugar stick is soluble in water.

VI. Wood is in the jungle. I can get sugar from the shops. Iron is very strong.

VII. You see, I can do this problem by measurement and experimentation. This is what I do in the laboratory.

VIII. I think I can solve this problem simply by seeing. How

can they have the same appearance? (Suppose so). Why should I suppose wrong things? I have never done it in science but in methematics I have done it.

IX. I think, I will do this and then that I will then tell (report) the final results to my teacher.

One comes across several such examples both in isolation and as supplementary stuff to the correct proposed tests. In the latter case, it appears to be an integral part of their thinking which only per haps Socrates can correct until the pupil considers himself quite stupid. In the absence of this approach, the data indicated that sixth grade boys have not given serious thought to this problem. Only 55 per cent among them have proposed one test, and the next thirty per cents only two tests. In the next higher grade, they were attracted to his extent of 45 per cent by the content rather than the form of the problem. This percentage then began to decline in the succeeding age groups approaching naught in grade X.

5. What does history of science tell us? It tells what attempts are made to seek answers to pinpointed questions in the prevalent frame of reference. Lord Boyle is credited to have performed one experiment and got two interesting inferences when he put his ticking watch into the evacuated receiver. He could see the watch but could not listen to its ticking sound. Why? Because sound needed a medium (air) whereas light did not, for in its propogation, it like magnetic attraction, was independent of it. To quote S. Lilley:

> Again, he put various burning things in the receiver—a lighted candle, burning charcoal, and so on—and he found that when the air was pumped out the fire was extinguished. In other words, combustion—burning—depends in some way on the presence of air. Then he tried putting small birds and animals in the receiver. When the air was pumped out these creatures found difficulty in breathing and they soon died. So air, he had proved, is essential to respiration and to life. Now he put these last two experiments together: burning depends on air; and breathing also depends on air, does it not seem likely that there is something in common between the two, that respiration and combustion are essentially the same process? Boyle put that suggestion cautiously; but we now know that he was quite right. In burning, the thing that burns combines with oxygen from the air. And in respiration, the oxygen from the air is carried by the blood from the lungs to different parts of the body, where it combines with other substances in a way that is essentially slow combustion. It took over a hundred years to make this clear, but Boyle had taken the first step [16].

It is necessary to understand the central message contained above. It is this: ideas and tests in science do not become immediately

acceptable. Issues may take a century or so before the problem is finally solved. The given test or hypotheses has to survive under cross circumstances. One has simply to trust the phenomenon or observation in the absence of any clearcut explanation or concept. Coming to our problem, milk and sugar go together. Sugar is soluble in milk. Put the three sticks in this solution or mixture. Taste the solution before and after the experiment. See what happens? The following inferences are possible:

(i) The one made of sugar is likely to dissolve.
(ii) The one made of wood will float.
(iii) The one made of iron will touch the bottom and if heavy, may break the glass container.

6. If sweetness of the milk has increased, the stick which is now dissolved was surely made of sugar—a sort of control experiment. Directly tasting the rods becomes now one of the acceptable confirmatory tests.

This shows that a given scientific observation or concept is true under certain conditions. It is under these conditions that the various tests separate out to meet the demands of the problem—for substitute for water exists—any other equally good solvent. A few pupils under study like little Lord Boyle have suggested a single test giving two inferences which have come from a couple of Grade IX, X pupils. Consider the following examples:

(i) I will chew. The hardest will be iron and the sweet tasty one will be the sugar one.
(ii) I will put the three sticks in the fire. The one made of iron will get first hot and then red hot. Others will be reduced to ashes. It may expand as well.

Lastly whereas they have suggested tests for distinguishing among three sticks, there does not appear any tendency to carry out intensive tests of the confirmation variety on any one of the sticks. The problem did not, specifically speaking, demand this behaviour but at the same time, did not bar its appearance also.

This problem is positively and significantly correlated with all the outside variables (with felt difficulty, it is negative and understandable) except the following with which it is insignificantly correlated: social adjustment, confidence in the problem, interest in the problem and generalisation to algebraic symbols (summation) implying thereby that the said four variables have not varied in relation to the proposing tests problem. Among significant correlations, in order of size, the problem correlates more with grade ($r=.6795$); using

summation (r=.7085); using proportion (.6798) beaker combinations (r=.5635); stating procedures (r=.5138); using insight (r=.5113); and failure to grasp the essence of the problem (reversible) than with the omnibus concept of intelligence (r=.3094).

17: FLOW OF WATER THROUGH A TUBE

Read the following introduction for the last problem to be done by you.

Background Illustration

Ram asked Mohan one day about the factors which assist in the drying up of the handkerchief. Ram replied that there are several answers to this question. Examples are:

1. The nature of the cloth, whether cotton, silk or wool.
2. Its colour.
3. Its dimensions: length, width and thickness.
4. Temperature.
5. Even weather.

Ram then replied that he now understood all the factors. But tell me through experiments.

Mohan then replied as follows:

If any one says that the drying up of the hanky depends upon its length, then I can suggest the following experiment. I will pick up three hankies of the same material (silk, wool or cotton), of the same width and thickness. But remember their lengths will be different. I will then wet them either under a tap or in a bucket full of water. I will then carefully spread them either in the sun or in the shade. If you object to this procedure, I can clip them on the rope also. I will then simultaneously time the experiment. Results can be somewhat as follows:

(1) All the hankies may dry up more or less at the same time. This means that length of the hanky is not an important factor.
(2) If the hanky of the largest length dries up first, I can then say that the length of the hanky is an important factor in the drying up process.
(3) If the hanky of the shortest length dries up first, I can then say that shorter the length, the quicker it dries up. If it is true, then the hanky of the largest length will dry up the last of all. Through experiments like these, I can test the influence or effect of other factors.
Now solve this problem.

The Problem

Have a look at the diagram given below:

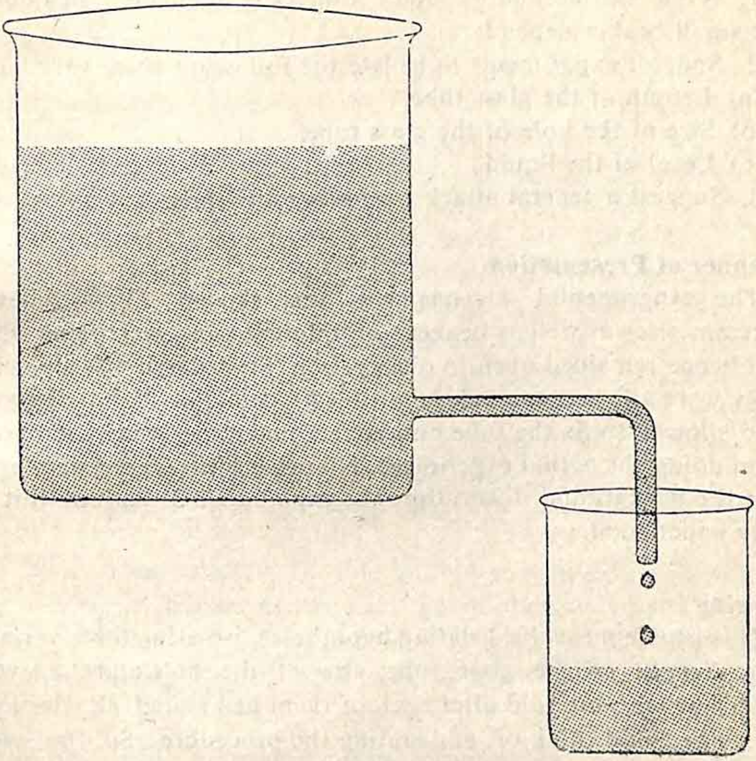


Fig. 14. Experimental set-up for Problem No. 17.

There are two beakers, one large and the other small. A glass tube is attached to the large beaker. The liquid from the large breaker flows through the glass tube and collects in the small beaker. Name all the possible factors upon which the filling in of the small beaker depends? Suggest experiments to test any three of the factors suggested by you.

Before you begin solving this problem, answer the following questions:

(a) Have you done a problem of this type? Yes/No

(b) Do you understand this problem? Yes/A Bit/No

(c) Do you find this problem difficult? Yes/A Bit/No

(d) Can you solve this problem? Yes/A Bit/No

(e) Do you find any difficult word in this problem?

If yes, write only those words.

Please solve the problem now.

1. Name all the factors upon which the collection of liquid in the small beaker depends.

2. Suggest experiments to isolate the following three variables:
(a) Length of the glass tube.
(b) Size of the hole of the glass tube.
(c) Level of the liquid.

3. Suggest a general attack for testing any three variables.

Manner of Presentation

The experimental arrengement was shown. Glass tubes of different sizes as well as beakers of different sizes were also shown and hence remained open to observation. Stop watch was also given. They were allowed to read the problem again and again. They were also allowed to fix the tube but, at the same time, were discouraged from doing the actual experiment. No other hints were given except that the background illustration was explained 2-3 times until it was fully understood.

Scoring

This problem involved stating hypotheses, isolating three variables only (length of the glass tube, size of the hole and the level of water) which were told after each of them had stated all the hypotheses he could think of, and stating the procedure. So the scoring for this question turned out to be:

Factor	Process No.	Score
Stating the factors	103	One mark each
Isolating the length	104	two marks
Isolating the size of the hole	105	two marks
Isolating the level of water	106	two marks
Stating the procedure partially	107	one mark
Stating the procedure fully		two marks

Elements and Aims of the Problem

Several alternative problems were considered. These were: the well-known simple pendulum problem; the Ingen hauz problem for determining conductivity of metals; factors affecting germination of

seeds and bending of rods. It was found that all these problems were either too time consuming or a bit simple, or a bit more abstract. Secondly, the present one was favoured because the flow of water through a glass tube appeared to fascinate school children. On reading the problem, it becomes evident that the problem is quite difficult because it involves excluding variables one by one by setting up control experiments after the variables have been identified, there being no limit on the number of hypotheses (or variables) to be proposed by the pupil at the beginning of the control experiment. These variables first to be clearly separated are: control of time, length of the tube, size of the hole and the level of water in the large beaker. There are other hypotheses or variables as well but the description of this problem only utilises these three variables. If not proposed, these were given when it came to their testing. This problem is, therefore, aimed at investigating the following:

(a) Up to what extent can the adolescent pupils set-up hypotheses?

(b) How is the solution of this problem attained? (To put in other words, up to what extent can the adolescent pupils clearly separate as well as test the three hypotheses by proposing experiments as well as stating the general method of attack?). To detail further:

(i) Time. It has to be kept constant for any set of observations or alternatively, the collecting beaker should be the same from experiment to experiment with a fixed reference mark or every time filled to the brim, a time consuming process. The presence of stop watch stimulates young pupils to think that it is of some use in the experiment.

(ii) Length of the tube. This involves selecting three glass tubes of different lengths but keeping size of the hole, as well as level of water constant.

(iii) Size of the hole. This, also, like the length of the tube involves choosing three glass tubes of the same length, keeping level of water the same but choosing three different sizes of holes.

(iv) Level of water. Here, the level of water in the large beaker is to be changed three times but keeping all other factors the same. For our purpose, it does not matter if a set of three different experiments is set up to isolate a particular variable, or the same experiment is repeated. in different contexts.

View on Hypotheses

They are like 'castles in the air'. Without them, it would have become impossible to build the Gigantic House of Science which always 'remains under repairs'. When set-up initially, they are neither right nor wrong. They simply suspect relationships among chance events occurring in this world and hence lend themselves to definitional difficulties with another added problem: they deal directly with the varied acts of psychic creations during encounters with problematic creations. Several terms taken from literature appear to provide some vague idea about their origin and nature: 'tentative ideas, tentative stabs, floating ideas, trial ideas, untested explanation, tentative assumption, shrewd guesses, generalisation, hunch, imaginative idea, mental tool, mental model, and building of temporary bridges'. No body knows how they arise in individual minds for the whole phenomenon is termed as retroduction. They, however, tend to make, depending upon individual worth, the opaque situations less opaque or transluscent. It then amounts to looking far or deeper into the phenomenon through a crystal ball. If they come out successful, they appear finally as grand generalisations or theories, providing, in the process, foundations for scientific research. Even then they do not become immortal as history of science shows for it is in the real nature of science to keep its possessions: facts, concepts and theory under continual gaze or open inspection. Out of the several competing hypotheses at the same time, the most powerful is considered the one which makes about half of the remaining hypotheses irrelevant on first experimentation. Incidentally, this cuts down the cost of experimentation as well. At the same time, either they answer the problem or alter it by making it more specific. If they do, they are then subject to the laws of logic (propositional reasoning, for example). If again successful, they still become more powerful concepts which either disturb the existing knowledge or finally merge with it beyond recognition over the years, without becoming the victims of value judgements as if behaving like neutrons. They continue to be characterised by unambiguity, simplicity, sharpness, pinpointedness, testability, conceptual clarity in the individual's head, and bordering on rightness-wrongness. To conclude, they are of doubtful reliability and validity which when subjected to test either clarify themselves or the problematic situation, generating in the process, new knowledge and skills. Their main functions are: to see the same problem from several viewpoints, thus providing a way out of the complex situation; suggesting new directions for experimentation; collection of

data for the problem under study; pinpoint strengths and weaknesses in argumentation especially when conflicting results appear; providing fruitful bases for investigation, making available in the process, new insights, concepts and skills hitherto unknown; predict the type of evidence needed to refute or support a given viewpoint; guide in selecting pertinent facts, concepts, direct as well as indirect evidence needed; and lastly, due to their very nature, to facilitate the emergence of insights and the highly varied acts of creation. So when seen in a limited context, there is a free exchange between the real and the hypothetical through a series of adjustments and readjustments of hypotheses that a new concept or skill, 'with no holds barred', is born like a baby which tends to seek perfection later on. The intensity of the struggle increases when a new concept or skill or techique tries to find a head-space in the well established body of knowledge and scientific methodology. It is in this context that the oft quoted statement of Prof. Oppenheimer becomes meaningful: it is the business of science to go wrong. To paraphrase it differently, it is the business of the human mind to go wrong or to get erratic in the search of new knowledge [17]. The problem under study just does that.

Presentation of Data

Table 3.17.1. Number of hypotheses proposed in terms of pupils gradewise as well as sexwise

S. No.	Grade	Sex	1	2	3	4	5	6	7
1.	VI	Boys	4	8	4	—	2	1	1
		Girls	1	2	9	7	1	—	—
2.	VII	Boys	4	5	9	1	1	—	—
		Girls	—	6	10	2	1	1	—
3.	VIII	Boys	5	8	6	1	—	—	—
		Girls	—	3	6	4	6	1	—
4	IX	Boys	—	—	7	9	4	—	—
		Girls	—	2	8	8	2	—	—
5.	X	Boys	—	9	4	3	8	2	1
		Girls	—	1	5	10	3	1	—
		Total	14	44	68	45	21	6	2

Table 3.17.2. Means and standard deviations gradewise as well as sexwise for the various sub-samples

S. No.	Grade	Sex	Mean	Standard deviation
1.	VI	Boys	0.00	0.00
		Girls	.05	.22
		Boys & Girls	.03	.31
2.	VII	Boys	0.00	0.00
		Girls	.10	.44
		Boys & Girls	.05	.63
3.	VIII	Boys	.90	1.84
		Girls	.6	4.62
		Boys & Girls	.53	3.43
4.	IX	Boys	1.20	1.33
		Girls	0.00	0.00
		Boys & Girls	.6	2.23
5.	X	Boys	.7	1.45
		Girls	1.4	1.63
		Boys & Girls	1.05	3.16

Summary of Results

It is possible to draw the following results from the above tables:

1. In all, the following eight hypotheses or variables were proposed out of which only the first three were supplied to the individual pupils, if not given by any one of them, for experimental testing:

(a) Length of the tube.
(b) Size of the hole quite frequently confused with the thickness of the glass tube, but later on, rectified.
(c) Level of water.
(d) Thickness of the tube.
(e) Height of the collecting beaker.
(f) Height of the beaker with no reference to the water column.
(g) Width of diameter of the beaker.
(h) Nature of the liquid.

2. Ignoring which one of the hypothesis is considered, it is safe to conclude that adolescent pupils are in a position to set up hypotheses. It is interesting to note that maximum number of hypotheses, seven in this case were set up by a boy each of grades VI and X.

78.5 per cent of the pupils have given 2 to 4 hypotheses Table (see 3.17.1). Large as well as small frequencies in the various cells indicate that hypotheses are set up over a wide I.Q. range not only within individual grades but also across the grades as well. The mean number of hypotheses given gradewise are:

VI	VII	VIII	IX	X
3.6	2.65	2.53	3.18	3.65

Except a small fluctuation in grades VII and VIII, the average number of hypotheses proposed remains almost equal throughout the grades. Whereas different hypotheses are set up by the adolescent pupils (seven in this study), the number of hypotheses per adolescent pupil remains more or less equal throughout the grades. This strengthens our earlier finding on proposing tests problem that the adolescent pupils do not suggest (or exhaust) all possible tests even when provided the opportunity, a finding inconsistent with Piaget.

3. Whereas the adolescent pupils are in a position to set up hypotheses, they also at the same time, contrary to Piaget's view, make comments in the form of arbitrary errors (see the sample responses which follow immediately). If true, their thinking is then tied to the experimental situation for one fails to see either specificity or clarity in their hypotheses. Perhaps, here is a case for providing opportunity to them to receive training in stating hypotheses for the very problem demands consideration in imagination the entire topsy-turvying of the experimental set-up with a view to obtain hypotheses for test experimentation.

4. When it comes to the testing of hypotheses (only three were included in this study), the overall performance is poor. Here, it is safe to conclude that the individual minds of the adolescent pupils have not yet become experimental for the scheme of thought (Testing hypotheses) is developing little across the grades. Boys and girls still try hard to equalise their whatever little performance throughout the grades.

In continuation, it is disturbing to point out that 81 per cent of the pupils could not test hypotheses at all. Except few successes on this problem, the ability to test hypotheses does not exist among pupils both boys and girls from grades VI to VIII. It equally does not exist among girls of grade IX in which only 50 per cent of the boys are in a position to attempt this problem. Interestingly enough,

girls surpass boys in grade X by 10 per cent, their respective percentages being 30 and 20, respectively. Taking an overall view, it is safe to conclude that majority of adolescent pupils fail to test hypotheses. The individual sexwise grade percentages in this context are mentioned below:

	VI	VII	VIII	IX	X
Boys	0	0	20	50	20
Girls	0	5	15	0	30

5. When it comes to the setting up of control experiments or isolation of variables turn by turn experimentally, only 27 pupils, 18 pupils and 4 pupils could isolate the following variables in this order: length, size of the hole and the level of water, the corresponding overall percentage being 13.5, 9 and 2. Strangely enough, when it comes to the testing of hypotheses, the variable which is more open to observation (level of water) turns out to be more difficult in isolation than the one which is less open to observation (size of the hole). Empirically, the three testable hypotheses under study can be ranked as follows:

1. Length of tube.
2. Size of the hole.
3. Level of water.

6. Out of nineteen pupils who could test hypotheses to a varying extent, very few of them could state or verbalise the methods of attack, the overall percentage ranging between 2 and 5 when those who suggested partial methods of attack were also included. But it is only in grade IX and X that the abilities to test as well as state procedures of attack appear to develop marginally enough as shown by the percentages given below:

Variables	IX	X
(i) Testing the length	22.5	25
(ii) Testing the hole	7.5	22.5
(iii) Testing the level of water	—	5
(iv) Stating the procedures:		
Partially	5	15
Fully	—	5

To digress, using the questionnaire approach and the well known Simple Pendulum problem on British children (15+), it was shown that those who omitted or could not attempt the problem were about 68.3 per cent. In the second part of the same study, using the same problem but following the individual mode of administration,

this percentage declined to 51.5 per cent. Among them (N=31), only over 19 per cent could isolate all the four variables (length, weight and volume of the bob and amplitude) at one go [18]. Since comparison between two cultures is deceptive, it is safe to conclude that ability to test hypotheses develops little even during later years of adolescence. The failure on this problem is too marked and hence needs to be mentioned. The sexwise failure in each grade is shown below:

	VI	VII	VIII	IX	X
Boys	100	100	80	50	80
Girls	100	95	85	100	70
Boys & Girls	100	97.5	82.5	75	75

7. As the performance on this problem is poor, one can easily say that the following behaviours elicited by this problem in terms of stating hypotheses, testing hypotheses and stating procedures of attack are failed (or manifested) over a wide I.Q. range not only within individual grades but also across the various grades as well.

Sample Responses

Let us mention below a few sample responses which trace the growth of the solution of this problem.

I. G-6, I.Q. 82, Grade VI, Score: Hypotheses — 3
Problem — 0

She looks at the experimental arrangement. Spends some time in explaining how the water is coming out from the larger beaker to the small beaker. She re-reads the problem. Appears to think for some time and then says that the factors are:

1. Length of the tube
2. Depth of the beaker (meant the height of the beaker)
3. The width of the beaker

Takes a metre rod and says that the length of the tube in 15 inches (in fact 15 cm). The length of the other tube is 10 inches (in fact 10 cms). In this beaker, 15 inch water comes. The length of the tube is also 15 inches. The width (circumference) is also 15 inches. The two beakers are connected by a glass tube. (On which factors, the filling in of the smaller beaker depends). Hesitates and then continues.

In the last experiment, we wetted the hankies. We put them in the sun. We then see which one of them dries up quickly. What did we do there?

We kept everything the same: length, colour, thickness and so on. Here, we must also do the same. Keep everything the same. Also you will find the clean water comes out of the small glass tube.

II. B-44, I.Q. 77, Grade VIII, Score: (i) Hypotheses — 3
(ii) Problem — 0

He glances at the experimental arrangement. He is quite happy that the water is coming out. He then sets the tube right. He then states the following variables:
1. Length of the tube
2. Size of the hole
3. Height of the large beaker
(a) Length of the tube. It should be less. Take a large beaker. Pour water (clean one) into this. Water comes out of the glass tube into the small beaker.
(b) Hole of the tube. The tube has a hole here and also there. Water passes through these two holes (wants to say that the tube is hollow). First of all, I took a beaker and then two glasses. I also took two tubes. I fixed one of the two in the beaker, water came out. I collected it in the glass. (Why the second glass?) How will I, otherwise, pour water into the beaker? Space is still available for writing. He must fill it in.
The drying up of the hanky depends upon several factors. Take hankies of the same colour. Their lengths, widths and thicknesses also the same. Take any utensil and put water into it. Wet these hankies. Drench the water out. Then spread out the hankies in the sun. They will dry up soon.
What are the factors?
1. To drench them
2. To spread in the sun
3. To place them in front of fire
4. Do not forget about the thickness of the hanky. It is very important.
(But you are not supposed to solve this problem) Sir, it is a similar problem. In place of hankies, there are glass tubes.

III. B-58, I.Q. 115, Grade VIII, Score: (i) Hypotheses — 3
(ii) Problem — 5

Looks thoughtfully at the experimental set-up.
Then says:
Water is coming out of the larger beaker into the small beaker. You know that water always flows from a higher level to another level. Hesitates for some time and then says:
Length is important.
If no length, no water will be collected. Laughs because the problem goes!
I mean length can change.
No! I mean hole.
It can be small, medium and big.
If the hole is large, water level in the larger beaker will fall down quickly. I can show you this, if you so wish.
(No, you need not).
The problem is how I can show this to you.
I will proceed as follows:

Take two tubes, one small and other large.
(Out of these tubes, which one will you pick up?)
I will pick up that one whose holes are the same.
Takes the two tubes. (Yes, you are correct).
If I have the large tube and fix it there, water will come out (yes).
If I fix the smaller one, the water will come a bit soon. There is going to be small difference in time. So smaller the tube, sooner the water will come out. (Now thinks! Then says that our problem is to find out whether length plays an important part in the amount of water collected).
Give me some time. Asks: What is this?
(This is a watch. I have told you earlier).
I am a fool because the water has to come out of the two glass tubes. Now, I think I can solve the problem like this.
Fix the tube as usual. Collect water for five minutes. Now change the tube. It should be either smaller or longer. Then collect water again for five minutes.
Now I can make the judgement like this.
If longer tube gives more water in five minutes, then length is an important factor.
If the shorter one gives more water to the collecting beaker, then I can say that length is an important factor.
If amount is the same, then length has nothing to do here.
Similarly, he gives experiments for isolating the factor of hole. Now he comes to the end of his wits. If I increase the size of the hole, the level of water falls down immediately. If I take the other tube (with a smaller hole), it takes time for the water to go. Level of water falls down slowly and slowly. (Level) Water fell in other experiments as well (while isolating the factors of length and hole). Thinks. Plays with the pencil. I think this factor has nothing to do because——water falls down any way. I think the first two factors are important.

IV. G-58, I.Q. 115, Grade VIII Score: (i) Hypotheses — 6
(ii) Problem — 5

She looks at the experimental set-up.
One must be careful in this experiment. See that the water is of right pressure. The tube should be kept straight. The hole should not be very small.
Then states the following hypotheses:

1. Length of the tube
2. Water: soft and hard
3. Width of the tube—thickness of the tube—size of the hole—tube with a hole.
4. Width of the beaker
5. Heigth of the beaker
6. Level of water.

In her case, one finds the economy of language while suggesting the experiment.

Take three glass tubes,one longer, one smaller and the third, the smallest. Fix the longest tube first.
Watch for water coming out into the smaller beaker.
Keep time for five minutes. Note water (level) in the small beaker.
Now take the second tube. Keep water level the same. Time for five minutes. Note the water in the small beaker.
Similary, with the third tube. Now I am to make judgement. If amount of water collected is the same in three cases, then length is of no use (Consequence). I must see with which tube, more water goes into the beaker for five minutes.
Similarly, she gives three experiments, quite acceptable for isolating the size of the hole.
She is again at the end of her reasoning. The pressure of water or level of water is important. If there is more water in the beaker, more water will flow into the smaller beaker. Similarly, it has less water and so less water will flow into the small beaker. From this, I conclude that pressure of water or level of water is important.

Discussion

The above sample responses trace the growth of the solution on this problem. These sufficiently indicate that the clear solution appears too late in the later years of adolescence. Why? Because there is failure to grasp the essence of the problem. So several arbitrary errors appear depending upon seeing the problem even when a short of complete solution is being given. It needs training to educate them out of these responses for they have little to do with the solution of the problem. Consider the following arbitrary responses.

(a) Way out of the problem followed by comments:
(i) Air exerts pressure. It is not necessary to use this apparatus. You are going to collect water. Why not use syphon?
(ii) You take a longer or smaller tube in length. I mean. See that the water flows from a higher level to a lower level.
(iii) Use a rightangle tube. The water will then move very smoothly. Do you see air bubbles?
(iv) The widths of beakers, lengths of glass tubes should be the same. Be careful that the tube is not kept slanting. You see that the water is coming out of the glass tube. Any way, the tube with a hole should be longer than the height of the beaker.
(v) You see the hole is there. Otherwise how can it flow? You see that the clean water is coming out. The presence of water is very essential in this experiment. Alternatively, the very presence of the glass tube is also essential.
(vi) I do not know. I find it difficult. I can skip this problem.

(b) Demanding information
What is the weight of the beaker? What is the length of the

glass tube? What is the size of the beaker? Tell me whether this water is hard or soft.

(c) Carryover effect or associativeness
Carryover effect from the last illustration.
Confusion between hankies and glass tubes.

(d) Failure to grasp the essence of the problem
Reproducing the question partially or fully. Repeating the question orally. Rephrasing and restating the question.

(e) Observing the experiment
Try to collect water. Trying to pour water. Taking interest in air bubbles. You see the lower beaker has no hole.

(f) Giving the information
Naming. This is a beaker or glass tube.
Describing. The beaker is round. Specifying the Dimension. The glass tube is 15 inches long.
Giving a formula and method not needed in the problem, Playing with the experimental material.

Incidence of Arbitrary Errors

It is difficult to count the arbitrary errors like match sticks for the various statements are either overlapping or are too connective. Considering only their dominant phases, their distribution gradewise is given below:

Table 3.17.3. Distribution of arbitrary errors gradewise

S. No	Errors	VI	VII	VIII	IX	X	
1.	Making comments	30	35	27	18	12	122
2.	Way out of the problem	10	16	12	8	7	53
3.	Observing and specifying, etc.	40	40	40	29	25	174
4.	Associativeness	4	5	2	1	1	—
5.	Demanding information	0	4	3	2	2	—
6.	Reproducing and repeating	30	35	32	22	18	137
7.	Miscellaneous	3	8	5	2	2	—
8.	Total	117	143	121	82	67	—

Seven distinct types of errors were noticed. Only four of them were found to be dominant, i.e., shared by more than 15 per cent of the pupils, their respective sharing being 61, 26.5, 87 and 68.5 per cent of the pupils. They arose naturally and more so when they were no longer within the borders of solving the problem. Each of the dominant errors as well as errors considered cumula-

tively suffers a hump in grade VII. One of such humps on the error: Making comments is shown below:

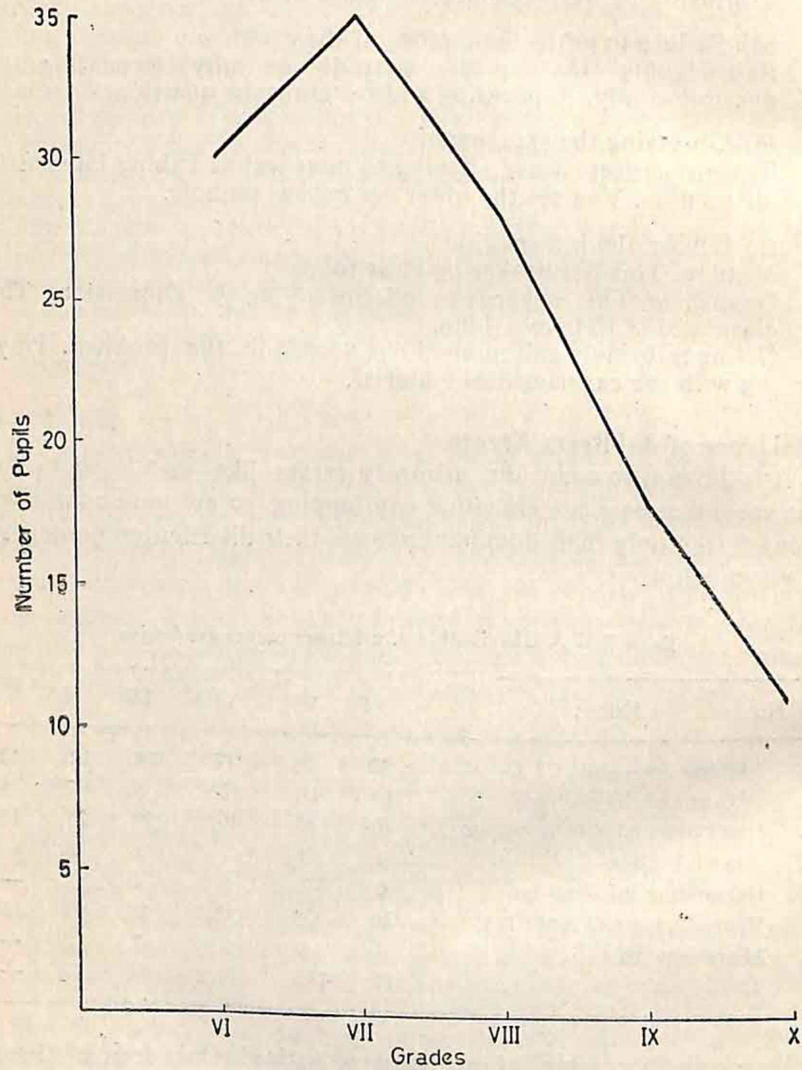


Fig. 15. Arbitrary errors (making comments) undergoing a hump in grade VII.

Errors when considered cumulatively also undergo a hump in grade VII.

Summary of Results

Let us know summarise the main findings on this problem.

1. Adolescent pupils are in a position to set up hypotheses but their mean number remains more or less the same across the grades. In continuation, adolescent pupils are not in a position to test hypotheses, the overall percentage being as large as 81 per cent. It is only in grades IX and X that the abilities to test as well as state or verbalise methods of attack appear to develop marginally enough.

2. Contrary to Piaget adolescent pupils are neither able to exhaust or to consider all possible hypotheses nor able to resist the temptation of making comments on the problem. If confirmed in other studies, it implies that their thinking is still tied to the concrete situation supported by another observation that one fails to see either specificity or clarity in their hypotheses.

3. This problem is failed by 81 per cent of the adolescent pupils which, in other words means that large scale failure takes place over a wide I.Q. range not only within individual grades but across grades as well with special reference to testing hypotheses and varbalising methods of attack.

4. Failure to grasp the essence of the problem results in all sorts of errors, seven categories were noticed out of which three were found to be dominant, each suffering a hump. When all the errors were added up, the cumulative frequency of these errors again suffered a hump.

5. Let us see how this problem correlates with other variables. For our interest there are two significant variables (Schemes of thought) involved in this problem, namely, stating hypotheses and testing hypotheses. Both are positively and significantly correlated with each other ($r = .158$).

Interestingly enough, both correlate more with grade than with intelligence. The former (stating hypotheses) correlates insignificantly with intelligence ($r = .1088$). Both correlate insignificantly with the following: home adjustment, health adjustment, social adjustment, emotional adjustment, school adjustment, understanding the problem, felt difficulty of the problem and interest in the problem as well as hotel problem and using two digits at a time. On the other hand, both correlate positively and significantly with several problems and schemes of thought. It is interesting to note that stating hypothesis is insignificantly correlated with the following problems: positive summation series problem, nine dot problem and formulating problematic situations (both fluency and flexibility),

Testing hypothesis is correlated positively and significantly with stating and verbalising method of attack.

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CHAPTER IV

THE MATHEMATICAL STRUCTURE UNDERLYING SCHEMES OF THOUGHT

Factor Analytical Study

It is one of the major objectives of any science to simplify knowledge. A highly mathematical technique called 'factor analysis' has recently become available, thanks to high speed computers, which extracts minimum number of factors that effectively explain data or the correlation matrix. They are easily interpretable, if properly hypothesised. But these factors, if they appear at all, are, generally speaking, difficult to interpret from the psychological angle for they are 'blurred averages' which reflect only the 'end products' of human thinking. Secondly, the individual factor loadings of the various tests included in the study on the significant factors properly rotated provide mathematical information about the behavioural composition of the included tests or variables; and are thus, the source of direct and concrete evidence of test validity. At the same time, they tell little about the processes which led to their development. Or to put in other words, 'what is actually developing' or 'what has actually developed' is not revealed. In a qualitative study of this type which attempts to investigate certain aspects of thinking during adolescence through the medium of problem solving, it was considered desirable to test the following hypothesis:

> Whether schemes of thought derived rationally from problems appear factorially when considered as outside variables along with the original problems from which they were derived.

A few more variables were also included not only because data on them were easily available but also they appeared to have varying degrees of associations with logical thinking. When seen in this

restricted context only, the component analysis was carried out on the combined matrix containing forty-five variables and the varimax rotated factor structures further examined in regard to the hypothesised structure. Keeping the same serial order number as already followed, these variables were:

1. Grade, i.e., length of schooling in complete years.
2. Intelligence.

3 to 7. Adjustment; Home adjustment; Health adjustment; Social adjustment; Emotional adjustment and School adjustment.

8 to 11. Immediate test reactions to the problems on presentation: Understanding the problem; Felt difficulty of the problems; Confidence in the problem; and Interest in the problem.

12 to 28. Problems. Seventeen in number

29 to 45. Schemes of Thought. Seventeen in number.

To reiterate, the basic objective to carry out this exercise was to explore the possibility of obtaining a few factors (if at all they could be obtained) and interpret them psychologically. In this exercise, it was further assumed that schemes of thought when pitted against problems would facilitate in former's factorial interpretation, the reason being the already known nature of the problems solved in the main part of the study.

Obtaining the Correlation Matrix

Using the above mentioned forty-five variables, a correlation matrix was obtained for the pooled sample (N=200) which was, later on, factor analysed, using Principal Component analysis for obtaining factors which were rotated by the varimax method of rotation [1]. Some of the significant features of this unwieldy correlation matrix are:

(a) Considering the half correlation matrix, and its mirror image, it is found on physical counting that it contains 990 correlations out of which 890 are positive and the remaining one hundred negative. Out of the 890 positive correlations, 652 are significant at one per cent level, 756 at five per cent level and 134 insignificant. Out of the 100 negative correlations, 7 are significant at one per cent level, 22 at five per cent level and 78 insignificant.

(b) Considering 198 correlations among variables relating to adjustment and immediate test reactions to the problems on presentation, and the variables included in the study, 144 are positive and 54 are negative. Out of the 144 positive correlations, 25 are significant at one per cent level, 112 at five per cent level and 32 are

insignificant. On the other hand, out of the 54 negative correlations, 8 are significant at one per cent level, 14 at five per cent level and 40 insignificant. Here special attention needs to be drawn in respect of one variable: "Felt Difficulty of the Problem" which was scored reversibly, i.e., higher the difficulty, higher the score.

(c) Considering 748 correlations among variables relating to problems and schemes of thought; and the variables included in this study, 747 are positive and only one negative. Out of 747 positive correlations 92 are significant at one per cent level, 693 at five per cent and 54 insignificant. The single negative correlation is significant at one per cent level.

(d) With occasional exceptions here and there, it is possible to interpret most of the correlations obtained among the forty-five variables included in this study. It is not possible as well as desirable to interpret each and every correlation on practical grounds. Selection of variables is, therefore, inevitable for immediate consideration. These few variables so judged are: grade, intelligence, emotional adjustment, felt difficulty of the problem, using constant difference, using summation, using proportion, beaker combinations, generalising to algebraic symbols (prorportion), testing hypotheses, formulating problematic situations (both fluency and flexibility) failure to grasp the essence of the problem and using insight. The correlations based on these variables only suggest the following tentative conclusions.

First, barring one correlation, all the problems and the schemes of thought are positively and significantly correlated with each other, exception being the negative correlation between using two digits at a time and using four digits at a time. It has possibly appeared because of failure to attack the problem from three aspects at the same time, a phenomenon closely akin to 'functional fixedness' (Karl Duncker) or seeing single aspect of the problem at a time as in phenomenology: problem solving theory as propounded by Van de Geer [2]. Secondly, grade (length of schooling) is positively and significantly correlated with all the schemes of thought and emotional adjustment. Whereas its correlation with intelligence is insignificant, it correlates negatively and significantly with the Felt difficulty of the problem. These correlations are understandable because intelligence even varying over a wide I.Q. range has remained the same throughout the grades while grade in turn has changed over 5 points only: 5, 6, 7, 8 and 9. The variable: Felt difficulty of the problem was scored reversibly. Correlation between Generalisa-

tion to algebraic symbols (proportion) and intelligence is insignificant because the former variable (scheme of thought) is developing little across the grades as seen empirically.

Secondly, there is insignificant correlation between intelligence and insight ($r=.126$). Even being positive, it has failed to reach the conventional five per cent level of significance. One should not be seriously disturbed at this finding because it implies that a given problem involving insight pervades over a vide I.Q. range. To put in simple words, a pupil of low I.Q. may solve the problem successfully whereas another one of high I.Q. may fail to solve it. The main study has confirmed this finding empirically.

Thirdly, there is a negative and highly significant correlation between Felt difficulty of the problem (due to reversible scoring) and emotional adjustment. Interestingly enough, emotional adjustment correlates insignificantly with Testing hypotheses and Formulating problematic situations (Flexibility), a sort of desirable supporting association or relationship usually imagined to exist. This study does not support this view. On the other hand, it means that both testing hypotheses and formulating problematic situations (flexibility) come naturally to adolescent pupils where the role of emotional adjustment, ofcourse desirable, is of little consequence. This finding is quite understandable in the light of Piaget's own studies in which he has quite cleverly sidetracked this problem [3].

Fourthly, the variables relating to the immediate reactions to the problems on presentation have, largely speaking, attracted a large number of insignificant correlations and, thus, have behaved unexpectedly. It is explicable on two counts: (1) problem orientation is highly individualistic in character and (2) blunt ratings on these variables have not been sufficiently discriminating enough. It is possible to interpret the whole correlation matrix like this quite meaningfully to a considerable extent on this principle: correlation is positive and significant when two variables are developing or varying in the same direction as the original one; it is negative and significant either due to reversible scoring or failure on pupil's part to tackle the combinatorial digital problem in its three aspects simultaneously; and it is insignificant when the given variable is developing little even across the various grades.

Obtaining the Factors

Correlations and factor loadings were computed by the Reading University Computer Unit, Reading, England. This was,

therefore, the justification of the Hotelling method used. The computer extracted 10 factors using Varimax method of rotation which were found significant because the ten latent roots of the correlation matrix, each taken separately, were greater than one [4]. The accumulated percentage of trace total variation accounted for by the first throughout the last component or factor has ranged from 35.409 to 72.755. There is another interesting observation as well. The percentage of trace total variation carried by the first factor is more than five times the amount carried by the remaining nine factors, each taken separately. Leaving the first factor, this possibly shows the existence of small group factors requiring hierarchical organisation not at all hypothesised in this study [4]. Lastly, before data on factor analysis are presented, it is necessary to simplify the apparent factorial complexity of the various variables or tests included in the battery by ignoring the small sizes of the various factor loadings. Here also, there is no uniform practice followed on judging how much small is small! Benjamin Fruchter has suggested that values of factor loadings less than .20 are, generally speaking, insignificant and hence, can be ignored [5]. This view is also shared by Cronbach. Anderson and Leton have ignored values up to .30 for the purpose of factorial interpretation [5]. In this study, all the factor loadings of the various tests on the various factors whose values are less than .25 numerically have been ignored. The remaining ones on the various factors have been arranged in the descending order for basing interpretation on larger and largest sizes of tests factor loadings on a given factor.

Table 4.1. Latent roots as well as percentage of trace total variation each component and accumulated percentage for the various factors

S.No.	Factors	Latent roots of correlation matrix	Percentage of trace total variation accounted	
			Each component	Accumulated percentage
1.	I	15.9342	35.4094	35.4094
2.	II	3.0471	6.7714	42.1808
3.	III	2.2534	5.0076	47.1884
4.	IV	2.2217	4.9371	52.1255
5.	V	1.9352	4.3004	56.4259
6.	VI	1.7208	3.8240	60.2499
7.	VII	1.5747	3.4993	63.7492
8.	VIII	1.4988	3.3305	67.0797
9.	IX	1.3621	3.0268	70.1065
10.	X	1.1916	2.6480	72.7545

Interpretation of Factors

Keeping in mind the main hypotheses being tested in this study through factor analysis, the various factors are now interpreted as follows:

First factor: It has high positive loadings on 25 variables. Except the factor loading of school adjustment on this factor (.279 which is low), all other factor loadings relating to the various aspects of adjustment and immediate test reactions to the problems on presentation are insignificant. Further, the following five variables have insignificant loadings on this factor:

1. Generalisation to algebraic symbols (Summation)		=	—.0560
2. Stating hypotheses		=	.1420
3. Testing hypotheses		=	.2030
4-5. Formulating problematic situations:	Fluency	=	.1908
	Flexibility	=	.1580

There is another interesting observation as well. Unexpectedly, the factor loading of intelligence on the first factor, of course, positive is found to be insignificant 0.1678. On the other hand, the loading of another variable, i.e., Grade is found to be on the high side (.7383). Seen concretely, this variable is tied closely to the secondary school years implying thereby that the various schemes of thought, twelve out of seventeen, are strongly dependent on it. And they emerge during secondary school years. It should not be lost sight of, that problems used in this study were solved gradually short of full completion in the closing grades only. Under the circumstances when factors are defined more by the inside qualitative analysis rather than by external features (form and material) as well as when intelligence is hardly found to differentiate during adolescene, secondary education ought to foster integration rather than differentiation of verbal as well as practical abilities in India (by implication) as in Britain (P.E. Vernon). It is a broad hypothesis which needs additional exploration through several factorial studies [6]. In this context, there is then no alternative left except naming the first factor as Schematic Learning General appearing during secondary school years. When a search for this type of factor is made in literature, it comes very close to complexity as propounded by R. Meili which means 'ability to realise complex intellectual structures' [6]. In his interpretation, he rejected g, V, N, or S because these arose from the very consideration

of the external characteristics of the tests included in various factorial studies by the past workers. P.E. Vernon, however, regards these factors of doubtful value, reason being his psychometric rather than developmental orientation towards the analysis of intelligence in the writer's opinion. While investigating the growth of the Schemata of Proportion during adolescence K. Lovell extracted a general component 'accounting for a little over 44 per cent of the variance' and termed it Central Intellectual Ability which encompassed the said schema' [7]. He further stated that the development of this schema is slow during the secondary years, a finding which is consistent with the finding of this study as well. But the Central Intellectual Factor is quite different from Schematic Learning General because the former is tied to general educational ability in the British tradition rather than the several evolving schemes of thought linked strongly with grade but little with intelligence as hypothesised in this study.

Second factor: Variables only relating to Adjustment have high loadings on this factor. The factor of "Felt difficulty of the problem", can be ignored (.2870). Only one problem (Rectangular cube problem) and one scheme of thought (Using two digits at a time) have significant loadings on this factor, the former being —.2765 (which can be ignored) and the latter —.3214 which is slightly above .30, the highest size so far ignored in the factorial studies [5]. So this factor can be interpreted as adjustment factor for the five tests reflecting this main variable have pretty high loadings on this factor. Their sizes are:

1. Social Adjustment = —.7619
2. Emotional Adjustment = —.7180
3. School Adjustment = —.7127
4. Home Adjustment = —.6765
5. Health Adjustment = —.6482

Third factor: This factor appears very clearly and dominantly and is closely akin to attitude towards "Problem Solving" or Problem Orientation. Intelligence, grade, adjustment and all problems as well as schemes of thought have insignificant loadings on this factor. The factor loading of the "Felt difficulty" variable is positive because it was scored in the reverse order. Secondly, its size is quite small and hence can be ignored (.2841). Other tests having significant loadings on this factor are:

1. Understanding the problem = —.8217

2. Interest in the problem = —.7614
3. Confidence in the problem = —.7593

This factor is, therefore, interpreted as Problem Orientation.

Fourth factor: This factor also appears quite clearly on which grade, intelligence, sixteen problems and fourteen schemes of thought, out of seventeen, each have insignificant loadings. The two main tests or variables having significant loadings on this factor demand on the part of the pupils, formulation of those questions whose answers they do not know as well as grasping the essence of the problem. It may be added that the type of question asked determined its category called flexibility in contrast to fluency in this study. On further reflection, this factor in one form or the other is closely related to creativity for it is being determined namely not only by the open problem but also by a series of contextual questions, whose answers little depend upon arithmetical rules learnt mechanically for they intentionally attract erratic thinking. Here, reference is to Problem No. 10 in which several questions intentionally invite wrong answers. It is also worth noticing that all tests or variables relating to problem orientation factor (already identified) and adjustment have insignificant loadings on this factor. The tests or variables which have highly significant loadings on this factor are:

1. Formulating problematic situations = —.9144
   Formulating problematic situations (Fluency) = —.9152
2. Formulating problematic situations (Flexibility) = —.7589
3. Grasping the essence of the problem = —.3378

Earlier, R. Meili, using similar approach but different tests and subjects, extracted two factors bordering on creativity which resemble the present factor under interpretation when seen from the angle of Central mental function underlying it:

1. Plasticity: the breaking down and reorganisation of structures. This seems to resemble Thurstone's second perceptual factor closely. (In our open problem, the subject goes on formulating and reformulating problems whose answer he does not know.)

2. Fluency: This corresponds to Thurstone's unrestricted or ideational Factor [6].

To reiterate, both the variables relating to stating hypotheses and tsteing hypotheses have insignificant loadings on this factor. This

factor is, therefore, interpreted as Sensing Problems Factor which is closely identifiable with 'plasticity in thought processes', fluency, or interest in generating difficulty problems (questions) yet in search of answers [6].

Fifth factor: It runs through the following variables: grade, seven problems and eight schemes of thought. Interestingly enough, it does not run through variables relating to both adjustment and problem orientation factors. It also does not run through two open problems: proposing tests and formulating problematic situations: fluency and flexibility (three schemes of thought also). The factor loading of grade on this factor (also the ninth factor yet to be interpreted) nearly reaches significance and hence has say in the interpretation of this factor. It throws a broad hint that the factor under study or interpretation is closely tied to the length of schooling which in this study is closely related to secondary education. In order to pinpoint sharply on this factor, consider below a few top loadings of six variables or tests on this factor:

1. Proportion problem	= −.7478
2. Generalisation to algebraic symbols (Proportion)	= −.7464
3. Beaker problem	= −.6697
4. Beaker combination	= −.6697
5. Using insight	= −.6259
6. Nine Dot problem	= −.5237

These tests require two schemes of thought in particular: proportion and insight (structuring and restructuring). In the proportion scheme of thought, generalisation of arithmetical operations to algebraic symbols in proportion context is involved. Side support to this consideration is further provided by Rectangular Cube Problem (−.3776); and Spring Balance Problem (−.3910). Lastly, whereas the scheme of thought related to Testing Hypotheses has insignificant loading on this factor, the other two schemes of thought (stating hypotheses and proposing tests) have just approaching significant loadings on this factor. All these considerations give indication that this factor is yet incomplete, that is, one sharp remove or step away from general generalisation of arithmetical operations to algebraic symbols but, at the same time, still inhering a very important scheme of thought: using insight. It is, therefore, interpreted as symbolisation which is closely identifiable with using algebraic symbols in proportion context or short of it, space or

visualisation (characterised by structuring and restructuring) factor as propounded by Thrustone [6].

Sixth factor: This factor also emerges clearly. It has quite a high loading on the Flow of Water problem (or the scheme of thought relating to testing and verification of hypotheses). Stating procedures or verbalising methods of attack variable has the second highest loading on this factor. The remaining fifteen schemes of thought as well as fourteen problems have insignificant loadings on this factor. The two problems still left (Rectangular problem and Counting rectangles maximally) have loadings of the same sign which just approach significance. The significant loadings on this factor are shown below:

1. Flow of water problem	= −.9174
2. Testing hypotheses	= −.9174
3. Stating procedures	= −.4861
4. Counting rectangles maximally	= −.3037
5. Rectangular problem	= −.2520

The problem: Counting rectangles maximally involves considering and reconsidering the same problematic diagram from as many different aspects or angles as possible. This factor is, therefore, interpreted as Testing Hypotheses which closely resembles verification of hypotheses keeping in mind the fact that methods of attacking the problem were given verbally without actually carrying out specific control experiments (Manipulation of experimental materials was allowed).

Seventh factor: This factor runs only through the problems and two schemes of thought. Particularly speaking, grade, intelligence, adjustment and immediate test reactions are the various variables which have insignificant loadings on this factor. Like the fifth actor, it closely lacks symbolisation, i.e, handling arithmetical symbols in a context characterised fully by the overall generalisation of using constant difference to algebraic symbols. The following significant loadings of the five tests support the above interpretation:

1. Positive summation series problem	= .7983
2. Hotel problem	= .5487
3. Generalisation to algebraic symbols (Summation)	= .5812
4. Using constant difference	= .4720
5. Negative summation series problem	= .4101

It appears that this structure is still evolving in relation to its

full generalisation to algebraic symbols. This factor is, therefore, interpreted as Using Constant Difference in arithmetical situations only.

Eighth factor: This factor runs through only four problems and three schemes of thought, seven in all out of thirty-four (17+17). Grade, intelligence and adjustment are the variables which possess insignificant loadings on this factor. When closely examined for relevance, these seven problems and schemes of thought possessing significant loadings on this factor hint at a factor which prevents pupils from taking into consideration all aspects of the problematic situation simultaneously. To put concretely, they consider any one of the aspects of the situation at a time resulting in stayput in thinking so far as onward progress of the solution of the problem is concerned. In this context, the following loadings may be considered:

1. Using two digits at a time	= —.6911
2. Using three digits at a time	= —.6671
3. Combinatorial digital problem	= —.6145
4. Using four digits at a time or seeing the digital problem as a whole	= +.01258

The factor loading of the last test on this factor is very insignificant. Therefore, this factor is interpreted as Aspect Character as propounded by Van de Geer in phenomenology in his theory of problem solving and is closely identifiable with either "fixedness" in problem solving or with a condition in which pupils try hard to exhaust all the possibilities only in one direction at the cost of the second aspect [2].

Ninth factor: It runs through grade, felt difficulty of the problem, three problems and three schemes of thought. Intelligence, adjustment and immediate test reactions to the problems on presentation are the tests which have insignificant loadings on this factor, exception being "the felt difficulty of the problem". In order of size, the factor loadings of the other tests which possess significant values on this factor are:

1. Using four digits at a time, or Seeing the digital problem as a whole	= —.6172
2. Felt difficulty of the problem	= +.5002
3. Height problem	= —.4783
4. Stating hypotheses	= —.4536
5. Using constant differences	= —.4263

6. Negative summation series problem = —.3659
7. Combinatorial digital problem = —.3392
8. Grade = —.3040

It is evident from above that seeing the problem as a whole has the highest loading on this factor. Whereas the variables: "Felt difficulty of the problem" has a moderately high loading on this factor (.5002), that of grade tends to approach significance (—.3040). The latter gives indication that the factor is tied slightly to grade (secondary education) depressed appreciably by the "Felt difficulty of the problem". When the eighth and ninth factors are considered separately, a hypothesis appears: before a problem is fully solved, it is first solved step by step with little concern or consideration of their order. Further, grade and felt difficulty of the problem influence the emergence of this scheme of thought to some extent. This, factor is, therefore, interpreted as "seeing the problem as a whole", which is closely identifiable with consideration of all possibilities dominating thinking, 'globalisation: uniting separate data into a single whole, an essentially creative capacity' (R. Meili); and 'seeing relations between and carrying out operations with numbers' (F.M. Earle, 6).

Tenth factor: This factor runs through intelligence (—.8277) and two problems as well as two schemes of thought. The factor loadings of the latter are:—.2830 (height problem); —.3698 (rectangular problem); and .3971 (beaker problem or combination). Further, variables relating to grade, adjustment and problem orientation have insignificant loadings on this factor. This factor, is, therefore, interpreted as Intelligence and has appeared probably because it formed the backdrop of the entire study.

Further Tentative Consideration of Overall Interpretations Made

About 10 years ago, N. Vaidya, using questionnaire approach as well as choosing problems more or less of the same type but limited in number extracted the following factors through Principal Components Method (Hotelling): attainment, practical, interest and adjustment. When the problems were reclassified in terms of schemes of thought, more appropriately, categorised performances by administering the same problems individually among similar group of fifteen years British pupils (N=31); only one factor of the general type appeared. The study, however, lacked both study population (N=60 and 30 in the main study) and hypotheses [9]. Nine years later, R.M. Misra, using Piaget type tests and one test identical to

one used in this study, formulating problematic situations, extracted the following factors through Principal Components Method: general adjustment, seeing the problem as a whole (combinatorial digital problem); interest in generating difficult problems (whose answers are not known to the 15 year old pupils studying science in a very good Hindi medium school in Rajasthan); and newness of the problem. He further concluded that pupils of this age fail to test hypotheses, i.e. their minds have not yet become truly experimental [8].

In this context, it is of interest to mention the work of R. Meili relating to the analysis of intelligence from the angle of Central Mental Function, the type of study badly needed in this area even today for it is more revealing than simply suggestive restrictedly as pointed out by Vernon [6]. In his framework, he rejected g as well as V, N & S, the factors usually reported in factorial studies for they were extracted what he termed as having judged from the external characteristics or features of the tests included in the various studies. Focusing on their inside and using Thurstone's Centroid technique, he extracted the following aspects of intelligence which fully describe the makeup of intelligence:

1. Plasticity: the breaking down and reorganisation, of structures. This seems to resemble Thurstone's second perceptual factor closely.
2. Complexity: ability to realise complex intellectual structures.
3. Fluency: this corresponds to Thurstone's unrestricted or ideational F factor.
4. Globalisation: using separate data into a single whole, an essentially creative capacity [6].

In the absence of the information available on the various tests used by him as the study was reported in French, above mentioned four aspects of intelligence are too general to mean anything. They need too many qualifying words before they can be pinpointed for extracting relevant meaning underlying these factors. For example, how to distinguish among plasticity, complexity and globalisation factors is anybody's guess. Specific naming of the obtained gross factors in his study is its major casuality despite using the developmental approach. Without resorting to factor analysis, F.M. Earle, using the Duplex series of Ability Tests, gave the following classification by comparing the results of numerous intelligence and other tests given at 10 to 13 years with the subsequent performance of the children at various types of secondary school course: "Knowledge

of words and comprehension of sentences, logical reasoning, seeing relations between and carrying out relations with: numbers and shapes; and comprehending the structure and functions of shapes, mechanisms and other objects, dealing with practical problems" [6]. This description is only suggestive lacking mathematical evidence as well as sharp meaning. Vernon has suggested here reanalysis of Duplex tests which yields scores on 'g' and other abilities [6].

The present study is also an attempt in this direction following the same line of thinking which attempted to investigate certain aspects of thinking during adolescence through the medium of problem solving in which intelligence varying over 50 I.Q. points is held constant throughout the various grades, the specific hypothesis being tested is: whether schemes of thought derived rationally from problems appear factorially when considered as outside variables along with the original problems from which they were derived on the principle that all thinking. processes which appeared to belong together should be put together by taking cue from the definition of intelligence as given by L.S. Hearnshaw: 'cluster of high grade skills concerned with problem solving' [6]. Tracing individual processes of thought across problems as well as grades and combining them into 18 schemes of thought, the following multiple group factors appeared:

1. Schematic Learning General.
2. Adjustment.
3. Problem Orientation.
4. Sensing problems.
5. Symbolisation.
6. Testing hypotheses.
7. Using constant difference.
8. Aspect character.
9. Seeing the problem as a whole.
10. Intelligence.

It is possibly due to the peculiar design of this study, i.e., matching of pupils on I.Q. points that the factor of intelligence has appeared at the end. To arrive at significant number of factors, Professor Cattell (1966) has suggested another technique called the 'Scree Test'. The Principal Component Numbers and their corresponding Eigen values are plotted against each other on a graph paper see Fig. 16. The resulting graph is usually curved which obtains linearity after suffering a kink. Those factors which form the straight line of the linear part of the curve are ignored or rejected on reasons that

'random and unique variance tends to swamp the common variance' [10]. Using this approach, the last two factors from the bottom upwards can be rejected [10].

It is of interest to ask at this stage which of the schemes of thought did not appear clearly in this factorial study? Unexpectedly, the answer is stating hypotheses (correlating significantly and positively with testing hypotheses) which has insignificant loadings on all the factors except one, i.e., seeing the problem as a whole. It appears that while following the current practice, the hypotheses which according to Piaget come naturally to adolescent pupils, are simply statements of observation and action, possessing negligible mathematical status. These may well appear at the preceding stage of mental development. It is a finding which is further substantiated by the consideration that whereas the first factor loading of this variable on the first factor is also negligible (.1419), the first factor loading of an equally inspired test: Proposing tests is not so (.7467). It may be added that it is a tentative finding which needs to be followed up. All the other schemes of thought in one form or another find their appropriate place in the above mentioned multiple factorial structure.

It is difficult to comment on the relevance or acceptance of these factors, for factors are considered "hypothetical mental constructs", "blurred averages", and "products of thinking" and even "unstable by their very nature". Like a stick, it is not difficult to break them into pieces and in turn each piece into sub-pieces. They further depend upon not only on the nature of tests used in the study, population tested but also the point of view held by the factorist. Even their naming is difficult because they are usually named more frequently on the basis of their external characteristics, features and appearance rather than the central mental functions or behavioural differences they embody. Their external forms and inside contents are hardly considered at the same time. Even when considered, they frequently fail to interpret the factors already in the offing. With little ingenuity, it is possible to extract as many factors or sub-factors as one wishes. If highly specialised tests are used, then even the well-known factors already well established in the field may not appear at all [6].

Using the same test results, the British and American psychologists interpret the same set of factor loadings on a particular factor differently. The use of suppressor variables as is the practice in the U.S.A. then, does not help for then indirectly, according to

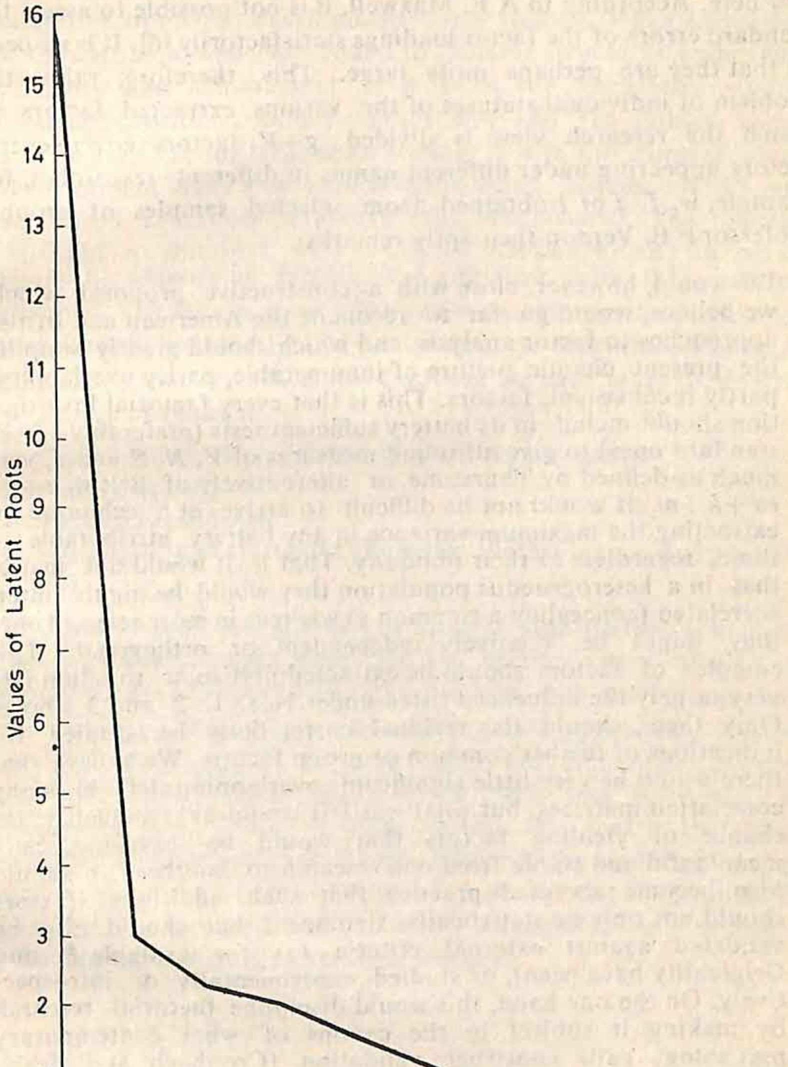

Fig. 16. Graphic relationship between Principal Component Numbers and their Corresponding Latent Roots.

Vernon, it tilts the balance in favour of 'g' [6]. The problem does not end here. According to A.E. Maxwell, it is not possible to assess the standard errors of the factor loadings statisfactorily [6]. It is suspected that they are perhaps quite large. This, therefore, raises the problem of individual statuses of the various extracted factors on which the research view is divided. $g+V$ factors cover several factors appearing under different names in different researches, for example, w, f, i or l obtained from selected samples of groups. Professor P.E. Vernon then aptly remarks:

> We would, however, close with a constructive proposal which, we believe, would go far to reconcile the American and British approaches to factor analysis, and which should greatly simplify the present chaotic picture of innumerable, partly overlapping, partly inconsistent, factors. This is that every factorial investigation should include in its battery sufficient tests (preferably agreed standard ones) to give all-round measures of V, N, S and I, very much as defined by Thurstone, or alternatively of British $g+v$: $ed+k$: m. It would not be difficult to arrive at a technique for extracting the maximum variance in any battery attributable to these, regardless of their obliquity. That is, it would not matter that in a heterogeneous population they would be highly intercorrelated (concealing a common g) whereas in more selected ones they might be relatively independent or orthogonal. This complex of factors should be extracted first so as to eliminate very largely the influenced listed under Nos. 1, 2 and 3 above. Only then should the residual correlations be studied for indications of further common or group factors. We believe that there would be very little significant overlapping left in many correlation matrices, but what was left would have a much better chance of yielding factors that would be psychologically meaningful and stable from one research to another. It should also become accepted practice that such additional factors should not only be statistically significant but should also be validated against external criteria (as for example P and Originality have been), or studied experimentally or introspectively. On the one hand, this would discipline factorial research by making it subject to the canons of what contemporary psychology calls construct validation (Cronbach and Meehl, 1955). On the other hand, it would encourage the development of tests with low *VNSI* content which would be more useful than those at present available for practical diagnostic purposes [6].

It is an excellent piece of advice backed up by practical experience but at the same time, it won't be irreverent to add that varied purposes of large and small scale studies should keep the varied modes of attack open. Fashion is necessary but not final. The studies of R. Meili and F.M. Earle are a pointer in that direction [6]. The

basic ideas of Gestalt psychology, Geneva School, J.P. Guilford, J.S. Bruner, Robert Gange and Suchman, etc., and others relating to acceleration of mental development under training when translated into testable hypotheses are bound to illumine structures underlying intelligence and achievement. It is from the qualitative studies done under controlled conditions that new and far sighted hypotheses may appear for large-scale testing. It is in this context alone that the present study was undertaken in which schemes of thought were derived from problems individually administered. The schemes of thought so obtained were analysed mathematically in which support for identifying factors was obtained from the problems themselves. The factors so obtained were specifically stated in their contexts which are closely tied to secondary education. The study simply shows that the Schematic Learning General lacks the following schemes of thought:

1. Generalisation to algebraic symbols (summation).
2. Stating hypotheses.
3. Testing hypotheses.
4. Formulating problematic situations (both Fluency and Flexibility).

The second and third schemes of thought indicate that the minds of our adolescent pupils have not yet become experimental. The first and fourth schemes of thought indicate that our pupils have not yet attained symbolisation as well as sensing problems, a factor very close to creativity. In case, there are difficulties in their measurement, the reason may well be that they have not yet developed in their minds. In other words, this means that either downgrade tests or search for these abilities in the higher age groups! Lastly, a very broad frame of reference is needed to answer questions which concern man and his education, occupation and aspiration in the closing half of the twentieth century.

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CHAPTER V

ANALYSIS OF THINKING PROCESSES, SCHEMES OF THOUGHT AND OTHER RELATIONS

Introduction

Thinking processes are not open to direct inspection or observation. Through the medium of problem solving, one can only hope to know certain key points which either map the route or trace the process of thought right from the moment the problem is presented to the moment it is finally solved [1]. In this chapter, we present the various processes of thinking as evoked in this study by analysing all the problems except the open ones regardless of their typology needed to solve them for the entire sample (N=200). The maximum frequencies for any one of the single thinking processes within any one of the grades as well as for all the grades put together are 40 and 200 respectively. Why this exercise of studying individual processes of thinking across problems as well as schemes of thought? The reason is simple—a single drop or pattern of drop is of major interest to us. It tells whether any particular thinking is present or absent in view of its being the maximum possible performance on it by the pupil as is assumed to be ensured by the individual mode of administering the various problems. However, the extent to which a particular thinking process is present is not shown but it can be safely inferred when all the thinking processes are seen together. The aggregate of usage of a process demonstrated by an individual may lead us to assess the degree to which the required concept or scheme exists. The data represented below for the eighty-seven thinking processes serially in the descending order of acquisition, each individual thinking process retaining its own serial number. With this step, the serial numbers of problems as well as those of the schemes of thought also stay intact.

Presentation of Data

Table 5.1. Sum total acquisition of eighty-seven processes of thought derived from the seventeen problems in the descending order of percentages, gradewise

S. No.	Process	Code	S. No. of contact problem	S. No. of schemes of thought	VI	VII	VIII	IX	X	Total	Percentage
1.	$10 \times ? = 50$	23	4	III	40	40	40	40	40	200	100
2.	$10 \times ? = 70$	24	4	III	40	40	40	40	40	200	100
3.	$10 \times ? = 100$	25	4	III	40	40	40	40	40	200	100
4.	$25 = ?$	7	2	I	35	35	38	39	39	186	93
5.	$? = 40$	9	2	I	34	35	38	39	39	185	92.5
6.	$? = 35$	8	2	I	32	35	38	39	39	183	91.5
7.	$40 = ?$	10	2	I	32	35	38	39	39	183	91.5
8.	$? = 50$	11	2	I	29	34	38	39	39	179	89.5
9.	$50 = ?$	12	2	I	28	34	38	39	39	178	89
10.	How much did they spend on vegetation	29	5	III	30	29	37	38	37	171	85.5
11.	Using beakers as weights	95	15	II	9	40	40	40	40	169	85.5
12.	How much did they spend on ice cream?	30	5	III	25	27	36	38	39	165	82.5
13.	How much did they spend on coffee?	31	5	III	25	26	37	38	39	165	82.5
14.	How much did they spend on bread?	28	5	III	29	26	31	34	39	159	79.5

15.	Using two digits at a time	50 (a)	9	V	36	30	30	23	40	159	79.5
16.	How far can a blind man see?	54	12	XVII	24	26	31	33	40	154	77
17.	Using four beakers at a time?	77	13	IV	14	31	35	35	38	153	76.5
18.	How much will it take for five men to cross the streams?	58	10	XVII	23	20	34	35	39	151	75.5
19.	How long will it take 10 boys to teach the school?	57	10	XVII	23	22	31	35	39	150	75
20.	How long will it take up to dry up 20 hankies?	56	10	XVII	22	19	31	35	39	146	73
21.	Counting rectangles maximally (first correct answer)	49 (a)	8	XVI	34	32	32	27	29	145	72.5
22.	Using both beakers and weights	97	15	II	0	14	40	40	40	134	67
23.	Joining the dots any way	66	11	XVI	9	16	25	39	40	129	64.5
24.	How much did they spend in all?	33	5	II	14	19	27	33	35	128	64
25.	How many birds left?	55	10	XVII	12	17	23	34	37	123	61.5
26.	How much did the manager return?	34	5	II	12	18	25	32	35	122	61
27.	What is the height of Mohan?	2	1	I	18	6	25	32	37	118	59
28.	How much had each to pay?	32	5	III	12	19	23	32	32	118	59
29.	Using four digits at a time	52 (a)	9	VII	6	9	29	34	40	118	59
30.	What is the volume of one rectangular cube?	46	7	III	18	20	17	25	37	117	50.5
31.	Joining the dots any way	67	11	XVI	8	7	20	39	40	114	57
32.	How many horns have eight donkeys?	63	10	XVII	24	28	21	18	22	113	56.5

S. No.	Process	Code	S. No. of contact problem	S. No. of schemes of thought	VI	VII	VIII	IX	X	Total	Percentage
33.	What is your name?	62	10	XVII	8	15	20	32	35	110	55
34.	What is the height of Sohan?	3	1	I	14	5	22	32	36	109	54.5
35.	Using three digits at a time	51 (a)	9	VI	15	8	26	19	40	108	54
36.	Joining the dot any way	68	11	XVI	4	7	18	34	40	108	54
37.	What is the height of Shyam?	4	1	I	14	6	21	31	34	106	53
38.	? = —30	15	3	I	6	11	16	33	40	106	53
39.	—30 = ?	16	3	I	6	9	16	33	40	104	52
40.	—45 = ?	17	3	I	2	9	16	33	40	100	50
41.	? = —55	18	3	I	2	9	16	33	40	100	50
42.	?	19	3	I	2	7	16	33	40	98	49
43.	?	20	3	I	2	7	16	33	40	98	49
44.	What is the combined apparent depths of tank when seen from bottom and top?	90	14	II	1	9	22	28	38	98	49
45.	How long is Shyam shorter than Mohan?	5	1	I	11	3	21	30	25	90	45
46.	Using one beaker at a time	74	13	IV	15	6	10	21	36	88	44
47.	How much time will it take to cut a 10 cm long stick?	60	10	XVII	10	1	8	12	27	58	29
48.	What is the apparent thickness of the fish?	92	14	II	0	0	13	17	26	56	28
49.	Using weights only	96	15	II	3	0	1	13	39	56	28

50.	What is the apparent depth of the tank?	87	14	III	0	0	11	17	26	54	27
51.	What is the thickness of the fish?	93	14	III	0	0	12	10	26	48	24
52.	Taking into account the weight of the beaker	101 (b)	15	I	0	2	0	18	27	47	23.5
53.	What is the distance in one round?	36	6	II	1	0	6	19	19	45	22.5
54.	What is the distance in four rounds?	37	6	III	1	0	6	19	19	45	22.5
55.	joining dots as asked	69	11	XVI	3	8	2	5	25	43	21.5
56.	What is the real position of the fish when seen from above?	80	14	III	1	2	14	5	20	42	21
57.	How many ducks are in all?	64	10	XVII	3	0	2	11	20	36	18
58.	How much time will it take to make four rounds?	39	5	III	1	0	4	10	14	35	17.5
59.	What is the real position of the fish when seen from below?	82	XIV	III	0	0	10	5	20	35	17.5
60.	Using two beakers at a time	75	13	IV	7	5	1	8	13	34	17
61.	The combined depth of fish when seen from top and bottom	83	14	II	0	0	9	5	20	34	17
62.	What is the thickness of the fish?	85	14	II	0	0	9	5	20	34	17
63.	What is the time taken to complete four rounds?	40	6	III	1	0	1	16	13	31	15.5
64.	Extension is proportional to the stretching force	98	15	III	0	0	1	4	26	31	15.5

S. No.	Process	Code	S. No. of contact problem	S. No. of schemes of thought	VI	VII	VIII	IX	X	Total	Percentage
65.	Suggesting calibration	99	15	—	0	0	1	4	26	31	15.5
66.	$10 \times ? = X$	26	4	IX	0	0	0	4	26	30	15
67.	$? \times 10 = Y$	27	4	IX	0	0	0	4	26	30	15
68.	To pick up the minimum number of hankies for a pair in a dark room	61	10	XVII	1	2	5	10	10	28	14
69.	Using three beakers at a time	76	13	IV	0	4	0	8	15	27	13.5
70.	Counting rectangles maximally fifth correct solution	49 (e)	8	XVI	1	4	3	6	12	26	13
71.	Isolating the length of the tube	104 (2)	17	XI	0	1	6	8	10	25	12.5
72.	Joining dots as asked	70	11	XVI	1	0	0	3	17	21	10.5
73.	Volume of the three cubes	47	7	III	1	0	0	6	12	19	9.5
74.	What is the final level of water reached?	48	7	II	1	0	0	6	12	19	9.5
75.	What is the name of fourth friend?	59	10	XVII	0	2	4	4	7	17	8.5
76.	Isolating the size of the hole	105 (a)	XVII	XI	0	0	6	2	9	17	8.5
77.	How far can the cow go?	65	10	XVII	1	0	1	7	7	16	8
78.	Counting rectangle maximally, third correct solution	49 (c)	8	XVI	2	3	2	3	3	13	6.5
79.	Joining the dots as asked	71	11	XVII	0	0	0	2	11	13	6.5

80.	X=?	13	2	VIII	1	0	0	5	5	11	5.5
88.	Counting rectangles maximally and correct answer	49 (b)	8	XVI	3	0	3	2	2	10	5
82.	—X=?	21	3	VIII	0	0	0	3	6	9	4.5
83.	? =—Y	22	3	VIII	0	0	0	3	5	8	4
84.	Counting rectangles maximally fourth correct answer	49 (b)	8	XVI	0	1	0	2	3	6	3
85.	Isolating the level of water	106	17	XI	0	0	3	0	2	5	2.5
86.	?=Y	14	2	VIII	1	0	0	2	0	3	1.5
87.	Taking into account time for rest	41	6	I	0	0	0	0	1	1	.5

Summary of Results

The main results of this analysis indicate:

1. The complex thinking processes arise from simple thinking processes. It is precisely for this reason that items relating to schemes of thought—generalisation to algebraic symbols, testing hypotheses, using insight and failure to grasp the essence of the problem—come out late in development. To strengthen this finding, it is further pointed out that it needs fairly well established schemes of thought relating to using constant difference, using summation and proportion before the relationship: Extension is proportional to the stretching force is discovered, largely speaking, by pupils of grade X (see at S. No. 64).

2. Whereas the study did not investigate the role of hints and cues in problem solving, an opportunity became available on the Nine Dot Problem, i.e., solution of the problem, using the top left corner was shown but not explained. What then is the overall performance on using the remaining three corners? It is found to be quite dismal for only 21.5, 10.5 and 6.5 per cent of the pupils were able to use the second, third and fourth corner successfully (see at S. Nos. 69, 72, 79).

3. In continuation, consider another educational problem of theoretical as well as practical importance. What happens when a given scheme of thought is incompletely developed? In this restricted context, consider the other two problems relating to making as many figures out of the four supplied digits (digital problem) and the beaker problem. The scheme of combinatorial grouping inheres those two problems. Incidentally, here also one illustration or hint each to start on the problem was supplied. Considering the two problems logically, it is safe to assume that the digital problem on the very face of it is more meaningful to the pupils than the beaker problem. The overall performance on these two problems indicated:

(i) When hint or illustrated step is given (67), using two digits at a time is done successfully by 79.5 per cent of the pupils. On the other hand, seeing the problem as a whole, in which all the individual digits are to be used in all possible combinations, the percentage of pupils drops to 59 per cent, a fall of 20.5 per cent. If judged by the frequencies alone, the gradewise frequencies favour the former rather than the latter, the sole exception being the frequencies in grade IX. There is a further decline of 5 per cent in overall performance in the case of using three digits at a time,

Hint appears to have favoured the formation of all possible two digit combinations only.

(ii) On another similar problem, i.e , relating to Beaker combinations, the given illustration (AZK) is not found of much use because they see the problem first as a whole before they return to three of its other aspects, namely, using one beaker at a time; two beakers at a time; and three beakers at a time. The corresponding extent of filling in the various categories in percentage are given below:

	Per cent
(a) Using four beakers at a time	= 76
(b) Using one beaker at a time	= 44
(c) Using two beakers at a time	= 17
(d) Using three beakers at a time	= 13.5

The question naturally arises: why has this happened? The possible explanation appears to be that the problem is first solved the way it is first seen with little concern shown to the supplied illustration. Attempt is then made to exhaust that aspect first; and if not satisfied, search for another line of inquiry is made. If judged within reach (or sense), effort is again made to exhaust that aspect. This process continues until the problem is reasonably solved (again a matter of individual judgement) in all its possible aspects as demanded by the problem. On the other hand, when the needed underlying scheme of thought is little developed, few responses across the various aspects of the given problem appear which, when seen cumulatively, tend to show poor systematic attack on it. To put the same point in other words, it means that when the same needed scheme of thought is fairly well established, responses under each category or aspect appear first before others under other categories are even considered. It is then safe to conclude that in the latter case, the problem was attacked systematically. There are good chances that, while doing this, a few responses may be missed but these are bound to be emitted in most of the cases on second or third reflection when majority of the responses are seen (or shown) in aggregate. Further, there are also good chances that quite a few responses may be repeated which have been called resting points in this study. These, in turn, again show the intensity of persistence with which a given problem is approached until it is fully solved. This effort appears to come naturally, only in this case. In the former case, even the presence of resting points does

not assist for then the problem is simply dropped in disgust as our experience showed.

4. Talking in a restricted sense, consider the case of another scheme of thought, namely, the testing hypotheses. It is a complex process when fully developed for it is only then that the individual minds of adolescent pupils become truly experimental. When both the content and form of the argument remain the same, first variable is isolated by 12.5 per cent of the pupils (see at S. No. 71). There is a sharp decline in this percentage when it comes to the isolation the second as well as the third variable, the two overall corresponding percentages being: 8.5 and 2.5 (see at S. Nos. 76, 85). The individual gradewise percentages are still more revealing as shown below:

Table 5.2. Percentages of pupils isolating the three variables gradewise

	VI	VII	VIII	IX	X
(a) Isolating the first variable	0	2.5	15	20	25
(b) Isolating the second variable	0	0	15	5	22.5
(c) Isolating the third variable	0	0	7.5	0	5

These percentages show that not a single pupil from grade VI is able to control any one of the three variables. Secondly, not a single pupil from grade VII is able to handle the second as well as the third variables. Thirdly, not a single pupil of grade IX is able to handle the third variable. It is only in grades VIII to X that ability to test hypotheses appears marginally among adolescent pupils as judged by the various percentages ranging from 0 to 2.5. It is of interest to mention that prior to the control of the third variable, several other processes of thinking pervade (or intervene) before the first variable is controlled successfully. Some of these processes when read from the bottom upwards are:

(i) Seeing as well as counting rectangles maximally by shifting attention from one line to another with a view to see another rectangle (process at S. No. 84).

(ii) Generalisation to algebraic symbols (processes at S. Nos. 80, 82, 83).

(iii) Joining the dots firmly as asked (process at S No. 79).

(iv) Careful reading of the problem including ability to suspend the answer if demanded by the test item (process at S. No. 75).

Using the same argument but now in the opposite direction, not less than seventy processes of thinking appear before the process relating to the control of the first variable appears at S. No. 71. Their careful reading shows that these relate to several schemes of thought.

Table 5.3. Number of processes underlying the various schemes of thought prior to the emergence of isolating the first variable of problem No 17

S. No.	Schemes of thought	No. of processes captured
1.	Using constant difference	17
2.	Using proportion	17
3.	Using summation	10
4.	Failure to grasp the essence of the problem	10
5.	Using insight	6
6.	Beaker combination	4
7.	Generalisation to algebraic symbols (Proportion)	4
8-10.	Using two, three and four digits at a time	1 each

The above mentioned frequencies, therefore, suggest that the first emergence of the testing hypotheses scheme of thought is fairly dependent upon some fairly well established schemes of thought.

5. It is possible interpret the whole table like this from the bottom upwards. In fact, one can begin at any serial number. The following outstanding conclusion emerges.

Complex problem solving processes arise from simple processes. If the mastery on the latter processes is weak, the complex process is not likely to be solved. In continuation, when any given scheme of thought is little developed, a few responses at the periphery of its structure appear. It is quite possible to increase their number, if a simpler problem, preferably concrete one but of the same type, i.e., inhering the same structure, is selected. Despite doing this, the scheme in its full form does not operate in full for all possibilities except a few directly open to observation are not filled in. This clearly implies that there is a limit beyond which a given problem can't be simplified at all hinting indirectly at the existence of the scheme of thought. This finding is in the nature of an hypothesis which when put in other words means that all processes of thought within the given scheme of thought as well as fairly well established

varied schemes of thought may constitute a scale or a continuum, really a grand hypothesis still in search of experimental proof under diverse conditions.

Analysing the Scheme of Thought

In this study, a scheme of thought is said to exist when thinking processes supposed to test the same mental functions across problems are aggregated categorywise. While considering the interpretations which immediately follow, it may not be lost sight of that different schemes of thought carried not only different maximum scores but also, the various thinking processes in majority of the cases were marked on 0 and 1 mark basis. Taking an analogy from the world currency, these seventeen schemes of thought may not have the same equal value, the problem of weightage to each being the inherent limitation of this study. Thirdly, with a view to overcome this impediment to some extent, the raws scores on the various schemes of thought were converted into T scores with means and standard deviations kept arbitrarily at 20 and 3 respectively. There was, however, a little justification for doing this, i.e., the ratio between the mean to the standard deviation was more or less equal to the ratio between the mean to the standard deviation of intelligence (Mean = 95.15: S.D. = 15.45). Below are presented means and standard deviations on the various schemes of thought in raw scores and T scores along with the gains of scores over the five-year period.

Summary of Results

What do the above data indicate? They indicate the following conclusions.

1. Except occasional fluctuations, the mean performance on the various schemes of thought shows an increasing trend with grade which is an expected finding. The close analysis of the various means indicates that some of the schemes are fairly well established; and others little established, the criteria for considering the same being their two respective means reaching within and across grades one fourth and three fourths of the maximum score on a given scheme of thought. Data so considered indicated:

(i) Five schemes of thought: Using constant difference, using two digits at a time, stating hypotheses, formulating problematic situations (fluency) and formulating problematic situations (flexibility) evolve continuously among pupils studying in grades VI through X. Among these schemes of thought, using constant difference is the

Table 5.4. Means, standard deviations and gains in scores per year in terms of raw scores for the various sub-samples, gradewise

S. No.	Scheme of thought	Max. score	Measure	VI	VII	VIII	IX	X	Gain in means from VI to X	Average gain per year
1.	Using constant Difference	19	Mean	6.88	6.88	10.53	14.18	15.70	8.82	1.76
			S.D.	7.00	8.30	7.94	5.56	16.17		
2.	Using summation	11	Mean	1.03	2.50	4.73	5.86	7.60	6 57	1.31
			S.D.	2.89	2.87	3.56	4.54	6.86		
3.	Using proportion	16	Mean	3.60	3.52	6.03	7.63	10.45	6.85	1.37
			S.D.	6.48	4.44	4.44	6.69	8.48		
4.	Beaker Combination	14	Mean	1.43	1.33	1.58	2.85	5.68	4.25	.85
			S.D.	2.62	1.97	2.72	4.03	6.80		
5.	Using two digits at a time	12	Mean	5.68	5.45	7.05	5.40	10.68	5.00	1.00
			S.D.	7.67	3.65	9.37	10.31	1.92		
6.	Using three digits at a time	24	Mean	3.75	2.18	6.40	4.60	11.90	8.20	1.64
			S.D.	7.58	9.93	13.12	12.32	10.58		
7.	Using four digits at a time	24	Mean	.65	.70	7.00	9.83	7.03	6.38	1.27
			S.D.	4.53	4.57	14.20	14.33	8.58		
8.	Generalisation to algebraic symbols (summation)	8	Mean	.10	0	0	.65	.90	.80	.16
			S.D.	1.25	0	0	3.73	3.79		
9.	Generalisation to algebraic symbols (Proportion)	4	Mean	0	0	.03	.50	2.7	2.7	.54
			S.D.	0	0	.31	2.65	3.68		

S. No.	Scheme of thought	Max-Score	Measure	VI	VII	VIII	IX	X	Gain in means from VI to X	Average gain per per year
10.	Stating hypotheses	Open	Mean	3.00	2.65	2.53	3.18	3.65	.65	.13
		(+7)	S.D.	2.72	2.69	2.84	3.10	2.55		
11.	Testing hypotheses	6	Mean	0.00	.05	.75	.60	1.05	1.05	.21
			S.D.	0.00	.62	3.43	2.23	3 69		
12.	Stating procedures	5	Mean	0	0	.75	.48	1.90	1.90	.38
			S.D.	0	0	2.22	1.61	4.49		
13.	Proposing tests	Open	Mean	1.10	2.23	3.00	3.83	5.53	4.43	.89
		12+	S.D.	2.14	1.94	2.45	3.22	5.20		
14.	Formulating problematic situations Fluency	Open	Mean	7.10	8.83	7.4	9.65	11.73	4-63	.93
		32+	S.D.	9.05	12.38	8.99	11.50	9.25		
15.	Formulating problematic situations Flex.	Open	Mean	3.78	4.08	3.68	4.05	5.25	1.47	.29
		12+	S.D.	4.13	5.84	4.18	3.77	3.76		
16.	Using insight	20	Mean	1.35	2.60	2.68	5.13	10.3	8.68	1.74
			S.D.	4.40	5.62	5.82	8.55	11.80		
17.	Failure to grasp the essence of the problem	17	Mean	3.65	3.00	5.83	8.05	10.03	6.38	1.27
			S.D.	4.59	5.10	4.85	6.23	5.56		

Table 5.5. Means and gains in scores from VI to X in terms of T scores gradewise for the various sub-samples

Sl. No.	Scheme of thought	VI	VII	VIII	IX	X	Gains in scores from VI to X
1.	Using constant difference	18.03	18.86	19.01	21.05	21.26	3.23
2.	Using summation	18.02	18.89	20.21	20.90	21.95	3.93
3.	Using proportion	18.83	18.80	19.01	20.10	21.86	3.03
4.	Beaker combination	18.56	18.44	19.88	20.03	23.90	5.34
5.	Using two digits at a time	19.22	19.07	20.12	19.04	22.58	3.36
6.	Using three digits at a time	19.25	18.68	20.24	19.55	22.28	3.03
7.	Using four digits at a time	18.86	18.86	20.51	21.26	20.81	2.05
8.	Generalisation algebraic symbols summation	19.04	19.01	19.01	20.9	20.18	1.14
9.	Generalisation algebraic symbols proportion	19.13	19.13	19.19	19.82	22.70	3.57
10.	Stating hypotheses	20.00	19.64	19.49	20.18	20.69	.69
11.	Testing hypotheses	19.43	19.49	20.30	20.12	20.66	1.23
12.	Stating procedures	19.22	19.22	20.15	19.82	21.56	2.34
13.	Proposing tests	18.26	19.22	19.88	20.57	21.91	3.65
14.	Formulating problematic situations Fluency	19.49	20.03	20.45	20.21	20.81	1.32
15.	Formulating problematic situations Flexibility	19.73	19.82	20.33	19.01	20.72	.99
16.	Using insight	18.92	19.34	19.40	20.27	22.13	3.21
17.	Failure to grasp the essence of the problem	18.74	18.41	19.85	20.98	21.98	3.24

only one scheme of thought which gets fairly well established in grade IX and X.

(ii) Another five schemes of thought: Using summation, using proportion, using four digits at a time, proposing tests, and failure to grasp the essence of the problem (reverse the meaning) evolve continuously among pupils of grades VIII to X.

(iii) Another two schemes of thought: Using three digits at a time and using insight evolve continuously among pupils of grades VIII and X (gap due to polarisation in thinking hypothesised as noticed in the experimental study) and IX and X.

(iv) Another three schemes of thought: Combinatorial grouping (Beaker combination), generalisation to algebraic symbols (Proportion) and stating procedures (or verbalising methods of attack) evolve only among students of grade X.

(v) The last two schemes of thought, namely, Generalisation to algebraic symbols (summation) and testing hypotheses evolve marginally in pupils studying in the closing grades.

2. Considering the initial acquisition of the scheme of thought in grade as well as its growth across the grades, the statistics presented in the table 5.4 indicate that there are eight schemes of thought which increase one or more than one mark worth per year; four increase ½ or more than ½ mark worth per year; two increase between a quarter and a half mark worth per year; and three which increase less than a quarter mark worth per year across the grades. When seen concretely, these gains per year are:

Table 5.6. Showing average gains made per year by the adolescent pupils on the various schemes of thought

			Per year
(i)	Using constant difference	=	1.76
(ii)	Using insight	=	1.74
(iii)	Using three digits at a time	=	1.64
(iv)	Using proportion	=	1.37
(v)	Using four digits at a time	=	1.27
(vi)	Failure to grasp the essence of the problem (reverse the meaning)	=	1.27
(vii)	Using summation	=	1.31
(viii)	Using two digits at a time	=	1.00
(ix)	Formulating problematic situations (Fluency)	=	.93
(x)	Proposing tests	=	.89

(xi)	Beaker combination	=	.85
(xii)	Generalisation to algebraic symbols proportion	=	.54
(xiii)	Stating procedures	=	.38
(xiv)	Formulating problematic situations (flexibility)	=	.29
(xv)	Testing hypotheses	=	.21
(xvi)	Generalisation to algebraic symbols (summation)	=	.16
(xvii)	Stating hypotheses	=	.13

The above gains per year on the various schemes of thought are quite dismal. In terms of T scores, the gains in scores over the five year period are equally dismal for they range from .69 to 5.34. These yearly gains tend to show that schemes of thought develop quite slowly from grade to grade or they are characterised by slowness, gradualness, laziness and labouriousness. If regarded as cognitive structures, it is hypothesised, they take time to develop during adolescence.

Sex Differences in Problem Solving

It became possible to investigate sex differences, if any, on Piaget type tasks in this study. The problem does not appear to be settled either way if one scans literature on sex differences relating to intellectual performance. Large scale intelligence testing has shown that both boys and girls score equally as judged by average scores [2]. Several studies from the highly industrialised countries have indicated sex differences in attainment in science (Shaycoft and Othess, 1963). Tyler has shown that at the primary as well as at the higher primary stage, the sex difference in regard to ability and achievement is small in comparison with the 'variation within groups of the same sex' [2]. Anastassi and others have shown that spatial, numerical and mathematical reasoning tests favour boys; and verbal and linguistic studies favour girls. According to the All India Survey of Achievement in Mathematics (NCERT, 1970), "boy showed superiority, over girls as per the expectations at all the three levels" [3]. It further added that in the case of the co-educational schools, the sex differences in fact get a bit diminished [3]. Sex difference, it is added, may, infact, appear due to 'within group phenomenon' [3]. A UNESCO publication: Science Education in Nineteen Different Countries mentions, largely speaking, marginal sex differences favouring boys in our country [4] and even this may

arise due to factors other than sex, for example, difficult test items and perhaps abundance of boys rather than girls in the study sample. To quote, the report:

> The contribution to the sex difference is much more pronounced in the physical sciences than in the biological sciences. Whether the causes of this sex differene are innate, which seem unlikely or whether they are the result of traditional practices in child rearing and formal education, it is impossible to say at this juncture. It is clear that here is a problem that deserves further attention if justice is to be done to girls in the field of science [4].

So here is a potential problem awaiting intensive investigation. It is necessary to indentify the impinging variables under diverse conditions: type of school, regional as well as cultural differences, and the role of educational variables such as recency of curriculum, innovative methodology of instruction and availability of instructional and illustrative materials, preferably of the programmed variety. Using 't' test, significant sex differences appeared on four problems and seven schemes of thought out of seventeen problems and seventeen schemes of thought each. In the main study, the sex-wise means across the grades on individual problems have shown that boys and girls try hard to equalise their performance as they move into higher grades. Keeping this experimental finding in mind, it is safe to conclude that, largely speaking, sex differences in problem solving, if any, are less prominent than they are supposed to be for they do not appear in most of the cases in this study. Other educational variables may intervene and show up these differences [5].

Successful and Unsuccessful Problem Solvers

It was one of the objectives of this study to determine the characteristic differences, if any, between the successful and the unsuccessful problem solvers. For this purpose, the scores of all the pupils (N=200) on all the problems except the two open ones (Formulating problematic situations and proposing tests problem) were added and arranged in the descending order. Top and bottom 25 per cent pupils were selected (N=50 each). It was found that the top group contained 33 boys and 17 girls, their respective percentage being 66 and 34. Interesting enough, the bottom group, on the other hand, contained 21 boys and 29 girls, their respective percentages being 42 and 58. This shows that boys dominate the top group

and girls the bottom group, their ratio of incidence being 1.82 and 1.36 in these groups respectively. Secondly, it was further attempted to test mean differences with the help of the 't' test on variables relating to intelligence, adjustment, immediate test reactions and seventeen schemes of thought. Considering the conventional level of significance, top group differed significantly at one per cent level from the bottom group in home adjustment, health adjustment, emotional adjustment, social adjustment, school adjustment and understanding of the problem. It equally differed significantly on all the seventeen schemes of thought, thus, providing indirect evidence in regard to validity for the existence of the various schemes of thought (in other words, they grow during adolescence). Lastly, teacher evidence when collected informally at the end of the tabulation stage indicated that most of the unsuccessful problem solvers are highly distractable, show poor concentration and are little interested in school work at all which was, however, not the case in the case of successful problem solvers.

References

1. Donaldson Margaret. A Study of Children's Thinking, Tavistock Publications, London, 1963.

2. (i) Quoted from Experimental Child Psychology by Hayne W. Reese & Lewis P. Lipsitt, Ibid. pp. 543-544.

(ii) Quoted from Sex Differences in Problem Solving by E.J. Sweeney, Stanford University, Stanford, 1953.

3. Quoted from All India Survey of Achievement in Mathematics, NCERT, New Delhi, 1970, pp. 113-119.

4. Comber L.C., and Keeves John P. Science Education in Nineteen Different Countries, Ibid. pp. 138-153; & 291-292.

5. It is only hypothesised for little is known about, sex differences in problem solving genetically.

CHAPTER VI

EDUCATIONAL IMPLICATIONS FOR SCIENCE TEACHING

Backdrop

Unlike countries, science has open frontiers. In our time, it has come of age: endless and faceless, stable and fluid, its knowledge exploding and imploding at the same time. It is a whole hearted brainy activity pricked on by intense curiosity in which scientific facts, concepts and insights emerge out of a series of imaginative adjustments and readjustments of hypotheses setting content and form all over the world. At home, its chief distinguishing characteristics are: dullness, drabness and drudgery for it is even today approached dogmatically in its entirety: teaching, telling, learning, experimenting and examining. It is as well controlled little by the practitioners of science. On the top of it, our young boys and girls like Atlas of Old are carrying a too heavy instructional load on their young shoulders with the consequence that they make little use of their talents and tools while still at school. When all these casual factors are aggregated, it is least surprising that educational anarchy does not result in the absence of educational visions, psychological structures and empirical investigations. Speaking restrictedly, the fundamental problem, therefore, for the psychologist is to investigate assumptions underlying human thought processes and the factors influencing them so as to provide guidance and information of the practical type to teachers with a view to enable them to work with confidence while teaching science to youngsters. The practitioners, really speaking, need to know of psychological foundations for the intellectual structures required in the subject of science itself and their methods of teaching throughout the school

course in order to effect a maximum of educational development. In other words, they need information about what is too difficult or too easy, what concepts they should be striving for to form in their pupils of a given age group. In science, especially they need to know how to motivate pupils by finding answers to these questions is not clear cut and is not likely to be defined by a field experiment like the present one. We, therefore, confine ourselves to those educational implications which emerge from this study.

Educational Implications

(i) Immediate reactions to the problem: These are highly individualistic reactions to problems on presentation and hence do set a tone for solving a given problem. Used as one of the outside variables, these have also appeared in our multiple factorial structure. This finding immediately directs our attention to the selection of suitable problems for promoting thinking among adolescent boys and girls. They should also be neither too difficult nor too easy. If they are too difficult, they are not likely to be solved, and this could have many undesirable consequences in the lowering of motivations.

(ii) Distinction between relevant and irrelevant variables: At the research frontier or when the problem is opaque to the problem solver, it is very difficult to distinguish between relevant and irrelevant variables. It is, therefore, necessary to choose only those problems whose solutions are unambigiously clear cut for they can be checked externally. The workable proposition appears to be that of analysing known problems in such a way as to develop research skills. In the beginning, problems may be used which involve only one variable. Examples are: What is the effect of length on the time period of this pendulum, and what is the effect of temperature on the germination process of this seed? It has been observed that the formulation of the pinpointed problems brings forth pinpointed thinking.

When experience like this are built in over the years, chances are very likely that the pupils will experience at first, the characteristics of scientific methodology. Some of these are: Consideration of the same problem from all the possible vantage points, trying hard to attack the problem first experimentally rather than theoretically, suggesting alternative solutions to the given problem, strengths and weaknesses underlying analogies, critical examination of the elements as well as the demands of the problem, guessing and tentative

solutions. If this end objective is kept in mind, mechanical acquisition of knowledge will be less prominent.

(iii) Use of programmed techniques: Our study has shown that the use of pinpointed questions helps students in thinking specifically. Here lies a possibility to lead pupils towards the solution of a problem by a series of well designed multifacet questions. For example, performance on the Fish problem could have been poorer had this not been done. It indicates that children can be helped to structure problems but among them, the only ones who succeed are those who possess the basic scheme or concepts needed to solve the problem available to them. It is only then that the concepts of 'Gestalt', 'whole' and the 'figure and ground' are brought under control and the concept of 'insight' is made to function without children knowing that they have actually solved the problem. Further, the study has also shown that hints and cues do not equally help all. These in themselves appear to be sub-concepts before the problem can be solved. To put in other words, it is not possible to simplify a given problem limitlessly.

(iv) Generalisation of arithmetical thinking to algebraic symbols: The study has shown that generalisation of arithmetical thinking to algebraic symbols comes out late in development. It has also shown that the individual minds of individual adolescents within the contexts of age, intelligence and problems chosen have not yet become truly experimental. The introduction of algebraic materials as well as experimentation necessitated as a result of hypotheses setting needs to be either tailored or even postponed, for we should not expect these pupils to reason formally about a problem underlying schemes of thought not yet developed. In fact, it might be argued that the learning of this type of material should be left to those who are going to be future mathematicians, scientists and engineers. For the rest, what can be tackled with success are use of 'life centred situations' or obtaining experimental solutions to abstract problems in day-to-day science teaching. This approach to science teaching may facilitate the transition from concrete behaviour to abstract behaviour during late adolescence.

(v) Extent of thought brought into play: The study has shown that adolescent pupils do think hard while engaged in problem solving. They view the problem as well as solve it in their own characteristic ways. The extent of thought brought into play both qualitatively and quantitatively is not strictly tied to a particular I.Q. point. But, in fact, it is manifested over a wide I Q. range. To

put in other words, it means that a problem or a thinking process is passed or failed over a wide I.Q. range not only within the individual grades but also across the grades as well. It is yet to be explored whether problem solving behaviour is related to personality traits. If so, teacher should not lose hope that a pupil with low I.Q. cannot really think hard on a problem. He should be sensitive to their individualistic modes of thinking and, in fact, encourage weak students to suggest solutions to the varied problems more frequently than it is usually done. This will build up their confidence in their own abilities. They will be thus educated out of their erratic responses.

(vi) Hump effect: In continuation, the study makes it abundantly clear that adolescent pupils commit a large number of errors while engaged in problem solving. These errors further increase when they ignore, forget or fail to grasp the basic requirement of the problem for they continue to rub the problem at the wrong end. Our experience with these problems showed that all these attempts were not blind at all. Whereas these errors finally decline with age, most of the dominant errors committed in several contexts suffer a hump each. This means that teacher should help his pupils see the same problem from multivaried aspects, let them make mistakes in the process, and if these are rectified or corrected, the evolving concept is going to be firm one with a 'broad measure of generality'. Naturally, children ought to be educated out of these errors which have either no business to be there or they are characteristics of the lower stages of mental development. It is a desirable educational goal to achieve diminution in responses from the bottom upwards in relation to the essential structure of the problem. Lastly, it is a moot point whether responses going beyond the essential structure of the problem be regarded as errors (problems in this study being a task oriented situations having a clear cut solutions) especially when the solution of the problem is not known to the psychologists.

(vii) Training to formulate meaningful questions: The study has shown that pupils ask all sorts of questions. Interestingly enough, some of these were too simple and others were not even questions at all. Contrary to Piaget, adolescent thought especially during the early years of adolescence was found to be dominated by the content rather than the form of the problem. They, therefore, need training in asking or formulating meaningful questions whose answers they do not know. It is possible because when seen cumulatively, all the acceptable questions fell into as many as

seventeen different categories, from increase of responses with age indicating the higher stage. They should have abundant opportunities to reverse their thinking—from concrete to abstract and back to concrete. They will be then trained to exhaust all possible combinations.

Attainment of Economy

Thinking along the lines suggested above will result in:

(a) Balanced emphasis on behavioural objectives of science education.

(b) Increment in stimulation, satisfaction, functional understanding and the development of skills, interests and appreciations.

(c) Improved methods of teaching and learning as the whole science content will become functional.

(d) The provision of rich and effective background preparation for diversified courses.

(e) Minimum stress on mechanical acquisition of knowledge.

(f) The probable development of independent habits of thought and action, self confidence, persistence and adjustment.

Lastly, in this type of educational environment, the children will be handling the process rather than the product of experience in a graded series of analysed and suitably selected science problems drawn from the significant areas of human living.

Problems for Further Research

The investigation of higher mental processes or functions is a complex task. Several general problems have been mentioned while evaluating the current status of research on human thinking in different paradigms. A few others which arise as a result of the present investigation are stated below:

1. It is necessary to psychometrise this test instrument. Once done, scores obtained from it can be related to other outside variables, intelligence, personality and creativity. Findings from these studies, it is expected are bound to illumine learning, perception and motivation.

2. It is necessary to replicate this study using—factorial design in which, at least, three levels of intelligence, socioeconomic status and achievement each are included. The findings so obtained can be compared with the findings of this study. A general pattern of findings having some sort of mathematical structure may emerge.

3. In the course of experimental investigation, hump effect in

different contexts has appeared. It is necessary to confirm or refute it by using different samples and problems.

4. A research question still remains: Do thinking processes develop uni-dimensionally? It is possible to analyse the same data using monotonic analysis trends.

5. The present study hypothesised the existence of schemes of thought and confirmed quite a few of them factorially. It, therefore, facilitates the investigation of hints and cues in the teaching learning process, a problem not yet clear.

6. It is also necessary to determine whether other schemes of thought exist. If they do, how are they related to each other? Do they possess any hierarchical organisation?

7. If meaningful information is made available on the internal characteristics of the various schemes of thought, a new exciting area of research in a big way may appear on the scene, i.e., acceleration of human thought under different conditions of training.

Lastly, the investigation of human thought in all its varied aspects is a complex task. So both problems and acts of problem solving are bound to appear refractory both to the psychologists and the method specialists. Their troubles are bound to increase still further when there is no unanimity in reported literature on the very definitions of thinking and problem solving or how an experimental situation is to be finally selected and approached for the very literature itself is little organised. But those difficulties, present to a varying extent in any business of research, need not deter the future investigators on thinking. Scientists discovered pencillin in the 'mouldy' bread. Research in educational psychology should also discover something similar. Over two thousand years ago, Plato talked of breaking the problem at its joints. The opportunity for that has just arrived for developing guidance ideas for the structural improvement of teaching learning processes.

THE GROWTH OF LOGICAL THI[NKING] IN SCIENCE DURING ADOLESC[ENCE]

Narendera Vaidya

Educational ideas have their very remote past traceable right back to ancient Greek thinkers like Plato and Socrates. Thus over the years todays educational philosophies and practices have steadily grown, Jean Piaget having a great influence on any cognitive study.

A study into "The growth of logical thinking in science during adolescence" as the title of the book implies is very imperative. It is important to know how the adolescent pupil does his logical thinking, choosing or eliminating processes as they get interpreted in his mind. The absense of this knowledge leads to teachers all over the world trying random shots in the dark of *where to strike* and perhaps succeeding only in stiffling the growing young minds and drawing them away from the experience and excitement of learning.

The book is divided into six comprehensive chapters. From basic concepts related to problem solving, the author has made a detailed study of various problems, giving actual examples. The mathematical structure underlying the various schemes of thought and thinking processes have been analysed and the conclusion highlights the educational implication for science teaching.

Rs. 15

OXFORD & IBH PUBLISHING CO.
New Delhi Bombay Calcutta